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Transient analysis of the DSIFs and dynamic *T*-stress for particulate composite materials—Numerical vs. experimental results

V. Guduru [a], A.-V. Phan [a,*], H.V. Tippur [b]

[a] Department of Mechanical Engineering, University of South Alabama, Mobile, AL 36688, USA
[b] Department of Mechanical Engineering, Auburn University, Auburn, AL 36849, USA

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ABSTRACT

The fracture behavior of particulate composite materials when subjected to dynamic loading has been a great concern for many industrial applications as these materials are particularly susceptible to impact loading conditions. As a result, many numerical and experimental techniques have been developed to deal with this class of problems. In this work, the fracture behavior of particulate composites under impact loading conditions is numerically studied via the two most important fracture parameters: dynamic stress intensity factors (DSIFs) and dynamic *T*-stress (DTS), and the results are compared with the experimental data obtained in Refs. [1,2]. Here, micromechanics models (self-consistent, Mori–Tanaka, ...) or experimental techniques need to be employed first to determine the effective material properties of particulate composites. Then, the symmetric-Galerkin boundary element method for elastodynamics in the Fourier-space frequency domain is used in conjunction with displacement correlation technique to evaluate the DSIFs and stress correlation technique to determine the DTS. To obtain transient responses of the fracture parameters, fast Fourier transform (FFT) and inverse FFT are subsequently used to convert the DSIFs and DTS from the frequency domain to the time domain. Test examples involving free–free beams made of particulate composites are considered in this study. The numerical results are found to agree very well with the experimental ones.

1. Introduction

Composite materials are particularly susceptible to impact loading conditions in which loads are applied rapidly, randomly or deliberately such as those seen in aerospace, automotive, production, or construction industries, etc. As a result, it is important to understand the fracture behavior of composite materials under these loading conditions. Conventional theories suggest that the fracture behavior can be assumed by the crack initiation, propagation and branching in the vicinity of a crack tip. The two important fracture parameters which can be used to describe the dynamic fracture behavior are the dynamic stress intensity factors (DSIFs) and dynamic *T*-stress (DTS). Therefore, many numerical and experimental techniques have been developed to study these parameters.

To determine the DSIFs and DTS experimentally, one needs to measure the crack tip deformations during a dynamic fracture event at high spatial and temporal resolutions. In the past the focus has been primarily with the accurate evaluation of the former although a few reports have addressed DTS measurements in recent years. The works which have focused on DSIF measurements include those of Dally [3] who used photoelasticity and high-speed photography. Tippur and co-workers [4–7] have used coherent gradient sensing (CGS) method to study the dynamic fracture behavior of a variety of composite materials. Kokaly et al. [8] have used moiré interferometry along with high-speed photography to study aluminum alloys.

The recent advances in high-speed digital imaging at recording rates exceeding a million frames per second has made it feasible to study fast-fracture events using other methods and improve the accuracy of measured fracture parameters. Recently, Kirugulige and co-workers [1] used the method of digital image correlation (DIC) and high-speed photography for the measurement of transient deformations near a crack under dynamic loading. In their work, random speckle patterns on a specimen surface before and after deformation are acquired, digitized and stored. In the undeformed image, subimages are chosen and the locations of similar subimages are identified on the deformed image. Once the subimages are found in the deformed image, the displacements can be easily estimated. The obtained displacements are processed using least-squares method to extract the DSIFs and DTS using asymptotic crack tip field expressions. The measurement of these fracture parameters are shown to favorably compare with their finite element analysis results. Moving from the photographic methods, Jiang et al. [9] used split-Hopkinson pressure bar apparatus to find the dynamic

* Corresponding author. Tel.: +1 251 460 7453; fax: +1 251 460 6549.
E-mail address: vphan@jaguar1.usouthal.edu (A.-V. Phan).

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responses in a pre-cracked specimen. They derived simple formula to find the DSIFs from dynamic responses using vibration analysis method. Ivanyts'kyi et al. [10] applied dynamic torsion on a cylindrical specimen with an external circular crack and proposed a combined numerical and experimental method to determine the DSIFs. Due to the significant influence of the *T*-stress on crack path and crack growth direction, more and more experimental methods devoted to the measurement of this parameter can be found in the literature. For example, Maleski et al. [11] measured the *T*-stress under both static and dynamic loading conditions by optimal positioning of stacked strain gage rosette near a mode-I crack tip.

On the numerical modeling side, several numerical methods have been developed to determine the DSIFs such as finite difference method (FDM, e.g., [12]), finite element method (FEM, e.g., [13]), boundary element method (BEM, e.g., [14]), symmetric-Galerkin boundary element method (SGBEM, e.g., [15,16]) and scaled boundary finite element method (SBFEM, e.g., [17]). In Ref. [15], boundary integral equations (BIEs) for elastodynamics in the frequency domain are employed in conjunction with a modified quarter-point crack-tip element and displacement correlation technique (DCT) to accurately calculate the DSIFs. A conversion of the frequency DSIFs to those in the time domain is done by using fast Fourier transform (FFT) and inverse FFT (IFFT) as described in Ref. [16]. Numerous numerical methods such as FEM (e.g., [18]), BEM (e.g., [19]), SGBEM (e.g., [20,21]), and SBFEM (e.g., [22]) have also been used for determining the *T*-stress, while SBFEM (e.g., [22]), SGBEM (e.g., [23]), and BEM (e.g., [24]) for the DTS. Phan [21] developed a non-singular boundary integral formulation based upon a stress correlation technique for evaluating the *T*-stress and the technique is extended to cover the DTS in Ref. [23]. Song and Vrcelj [22] extended the SBFEM to evaluate the DSIFs and DTS for two dimensional problems without any requirement of internal mesh and asymptotic solution close to the singular point. Sladek et al. [24] used BEM to express path independent integral (*M*-integral) formulation through the dynamic *J*-integrals for determining the DTS on the basis of relation found between the *M*-integral and *T*-stress. A comparison was conducted to the DTS values computed by the *M*-integral, boundary layer and displacement field methods for a rectangular plate with a central crack.

BEM have been recognized as an effective technique for fracture analysis (e.g., [25]). The key feature of the BEM is that only the boundary of the domain is discretized. This implies that, for fracture analysis, the singular stress field ahead of the crack is not approximated, and that remeshing a propagating crack is an easier task. Among the variants of the BEM, SGBEM (e.g., [26]) has several additional advantages: (a) its coefficient matrix is symmetric as the name implies; (b) the use of both displacement and traction BIEs enables fracture problems to be solved without artificial sub-domains; and (c) unlike most variants of the BEM, standard continuous elements can be employed. Thus, SGBEM can easily exploit highly effective quarter-point quadratic elements (e.g., [27]) to accurately capture the crack tip behavior.

In addition to important developments of the SGBEM for stress and fracture analysis in elastostatics, the SGBEM for elastodynamics in the Fourier-space frequency domain has recently been extended to fracture applications [15,16,24]. Following an SGBEM fracture analysis in the frequency domain, FFT and IFFT are subsequently employed to convert the DSIFs and DTS from the frequency domain to the time domain. These transient responses, especially in the immediate aftermath of an impact loading, are of special interest as most dynamic responses usually reach their maximum value during this period.

In this work, the SGBEM for elastodynamics in the Fourier-space frequency domain developed in Refs. [15,16,24] is employed to evaluate the DSIFs and DTS for free–free beams made of particulate composite materials. To this end, the effective material properties ultrasonically measured are utilized. These numerical solutions are compared with some known experimental results for the purpose of validation.

2. Fracture analysis using SGBEM

In this section, fracture analysis using the SGBEM for 2-D elastodynamics in the Fourier-space frequency domain is briefly presented. More details of the technique can be found in, e.g., [16].

Consider a domain of boundary Γ containing a crack. Let $\Gamma = \Gamma_b \cup \Gamma_c^+ \cup \Gamma_c^-$ where Γ_b is the boundary of the non-crack part, and Γ_c^+ and Γ_c^- are the "plus" and "minus" surfaces of the crack, respectively. Further, let $\Gamma_b = \Gamma_{bu} \cup \Gamma_{bt}$ where Γ_{bu} and Γ_{bt} are the boundary parts where displacement and traction are known, respectively. Finally, let $\Gamma_t = \Gamma_{bt} \cup \Gamma_c^+$ and note that traction is assumed to be known on the crack surfaces.

For a given angular frequency ω and a source point P interior to a 2-D domain of boundary Γ, the displacement BIE for elastodynamics in the frequency domain is given by

$$\mathcal{U}(P,\omega) \equiv u_k(P,\omega) - \int_{\Gamma_b} \left[U_{kj}(P,Q,\omega)t_j(Q,\omega) - T_{kj}(P,Q,\omega)u_j(Q,\omega)\right] dQ + \int_{\Gamma_c^+} T_{kj}(P,Q,\omega)\Delta u_j(Q,\omega) dQ = 0 \quad (1)$$

where Q denotes a field point, U_{kj} and T_{kj} are the elastodynamic kernel tensors (e.g., [28] or [16]), u_j and t_j are the displacement and traction vectors, respectively, and Δu_j is the crack displacement jump vector.

When P is off the boundary, U_{kj} and T_{kj} are not singular and it is possible to differentiate Eq. (1) with respect to P, resulting in the displacement gradients. By substituting these gradients into Hooke's law and then Cauchy's relation, one gets the following traction BIE:

$$\mathcal{T}(P,\omega) \equiv t_k(P,\omega) - n_l(P) \int_{\Gamma_b} \left[D_{kjl}(P,Q,\omega)t_j(Q,\omega) - S_{kjl}(P,Q,\omega)u_j(Q,\omega)\right] dQ + n_l^+(P) \int_{\Gamma_c^+} S_{kjl}(P,Q,\omega)\Delta u_j(Q,\omega) = 0 \quad (2)$$

where n_l is the outward normal vector to the boundary and the elastodynamic kernel tensors D_{kjl} and S_{kjl} can also be found in [28] or [16]. This traction equation is required for dealing with crack geometries.

It is known that the limits of the integrals in Eqs. (1) and (2) exist as P approaches Γ. From now on, for a boundary source point P, the displacement and traction BIEs are understood in this limiting sense.

Unlike the collocation methods, the Galerkin approaches enforce Eqs. (1) and (2) over the entire boundary. To obtain a symmetric coefficient matrix as the name SGBEM implies, Eq. (1) needs to be enforced over Γ_{bu} while Eq. (2) is enforced over Γ_t. This is done by using the shape function ψ_m, employed in approximating the boundary tractions and displacements, as weighting functions for these equations

$$\int_{\Gamma_{bu}} \psi_m(P)\mathcal{U}(P,\omega)dP = 0 \quad (3)$$

$$\int_{\Gamma_t} \psi_m(P)\mathcal{T}(P,\omega)dP = 0 \quad (4)$$

The main computational task in numerically implementing Eqs. (3) and (4) is the evaluation of their singular integrals.