Engineering Analysis with Boundary Elements 34 (2010) 97–104

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

One method for electric field determination in the vicinity of infinitely thin electrode shells

Nebojsa B. Raicevic *, Slavoljub R. Aleksic

Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, P.O. Box 73, Serbia

ARTICLE INFO

Article history:
Received 6 July 2009
Accepted 26 August 2009
Available online 27 September 2009

Keywords:
Electrostatic fields
Equivalent electrodes method
Cable terminations
Strip lines
Characteristic impedance

ABSTRACT

One numerical method, the so-called equivalent electrodes method (EEM), has been applied to electric field calculation at planar and axially symmetric, extremely thin electrodes. The results obtained for characteristic impedance and effective relative dielectric constant of strip lines are presented, as well as electric field and potential distribution at geometrically modelled cable terminations. The accuracy of results of EEM depends on the number of used equivalent electrodes (EEs) and the error is inversely proportional to the selected number of EES. The method is very simple and exact and in limit process, with the number of the EES, gives exact results. However, in calculations certain problems may appear because of the increase EEs number, especially at very thin electrodes. The electric field being extremely high at the sharp electrode's ends, compared to other parts of the electrode, so it is possible to use smaller and denser EEs. The improvement proposed in this paper enables accurate calculation using a smaller number of EEs. Since the density of equivalent electrodes is higher, equivalent radii are smaller and EEs are not set at the same distances. The number of EEs in other regions is reduced. Some results obtained by the application of EEM have been compared with values obtained using the finite elements method (FEM) and very good matches have been identified.

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1. Introduction

For a long time, most of the current numerical methods [1–3], including finite element method (FEM), have not been efficient in analyzing the electromagnetic fields of very thin electrodes, the examples of which may be the problems of thin deflector at cable terminations and joints [9–12], perfect conducting plates at strip lines [13,16], thin floating stripes for electrostatic field modelling [14,17], etc. The accuracy of boundary element method (BEM) [2,3] depends on the accuracy of the evaluation of nearly singular integrals [6,7], especially for thin domains. When load points approach the boundary, it is a limiting process that the distances between the source points and the field points tend to zero. The conventional integration methods are not available for computing the interior quantities, close to the boundary.

Equivalent electrodes method [8–14,16,17] is a numerical method for approximate solving of non-dynamic electromagnetic fields and other potential fields of theoretical physics. In the formal mathematical presentations, EEM is similar to the method of moments (MoM) form [1] and the fast multipole boundary element method [4,5], but it is very important to emphasize the difference in physical fundaments and in the matrix forming procedure. In recent years, more and more attention has been paid to researches into the meshless methods, which makes them a popular direction of computational electromagnetics. As the meshless method, EEM has some advantages over the traditional computational methods. Also, EEM application does not require numerical integration of any kind, in contrast to the MoM application, which always requires numerical integration, and thus produces some problems in the numerical solving of non-elementary integrals having singular subintegral function. As a numerical method, EEM is applicable for all system geometries.

The moving least-squares (MLS) approximation is one of the bases of the meshless method. A disadvantage of the conventional least-squares method is that the final algebra equations system is sometimes ill-conditioned. This problem exists in the case of infinitely thin electrode shells. EEM is insensitive on sharp electrode edges, but the number of EEs can be multiplied. The primary disadvantage of the EEM is that the coefficient matrices are full. It can be exceeded using appropriate equivalent electrodes.

In the case when the system has several electrodes, or when the multilayer medium exists [13,14], it is convenient to use Green's functions for some electrodes, or for stratified medium, and after that the remaining electrodes substitute with EEs.

Because of the combination of both numerical and analytical determination, this is one hybrid method. It is significant to

* Corresponding author. Tel.: +381 18 529 447; fax: +381 18 588 399.
E-mail address: nebojsa.raicevic@elfak.ni.ac.rs (N.B. Raicevic).

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doi:10.1016/j.enganabound.2009.08.002

emphasize that the EEM computer time calculation is shorter than the computer time of some other methods.

2. Theoretical background

The basic idea of the proposed theory is that an arbitrary shaped thin electrode can be replaced by a finite system of equivalent electrodes (Fig. 1).

Depending on the problem geometry, flat or oval strips (for plan-parallel field) and spherical bodies (for three-dimensional fields [23]), or toroidal electrodes (for systems with axial symmetry) can be commonly used. In contrast to the charge simulation method, when the fictitious sources are placed inside the electrodes volume, the EEs are located on the body surface. The radius of the EE is equal to the equivalent radius of electrode part, which is substituted. Also the potential and charge of the EE and of the real electrode part are equal.

So it is possible, using boundary condition that the electrode is equipotential, to form a system of linear equations, with unknown charge densities of the EEs. Solving this system, the unknown charge densities of the EEs can be determined and then, the necessary calculations are based on the standard procedures [18,19].

Some examples are selected to demonstrate the applicability of the EEM for two-dimensional potential problems. The results obtained for these examples are compared with the existing solutions that have been published in the literature [9–19]. In order to illustrate the application of EEM in electrostatics, for electric field determination in the vicinity of infinitely thin electrode shells, several examples for plan-parallel and axially symmetric electrode system will be presented in the following text.

The procedure for equivalent radius (ER) determination will be shown in more details on the example of a thin strip line of great length and width d. The ER of the observed strip line is linearly proportional to strip width d and can be presented in the form of $A_e=kd$, where k is the proportionality factor. Since it is already known, the exact value of this factor is $k=0.25$ (Appendix I).

When applying EEM, and because of the existing symmetry, a strip conductor will be divided into $2N$ auxiliary shells of equal width, $\Delta x=d/2N$, as in Fig. 2, and then, they will be replaced by cylindrical EE of radius $a_e=k\Delta x$.

Since the axes of these equivalent electrodes are located on $x=\pm x_n$, $x_n=(2n-1)\Delta x/2$, $y=0$, $n=1,2,\ldots,N$, the potential can be approximately expressed in the form

$$\varphi=\varphi_0-\sum_{n=1}^{N}\frac{q'_n}{2\pi\varepsilon}\log\sqrt{[(x-x_n)^2+y^2][(x+x_n)^2+y^2]}, \qquad (1)$$

where q'_n, is the line charge of auxiliary, i.e. EE, and φ_0 is the additive constant. The total strip line charge per unit length is

$$Q'=2\sum_{n=1}^{N}q'_n\,. \qquad (2)$$

From the condition that the EEs are of equal potential and have the same potential as the strip line, U, the following system of linear equations is obtained, where the line charges of EEs are unknown

$$U=\varphi_0-\sum_{n=1}^{N}\frac{q'_n}{2\pi\varepsilon}\log\sqrt{(x_n+x_m)^2[(x_n-x_m)^2+a_e^2\delta_{nm}]},\ m=1,2,\ldots,N, \qquad (3)$$

where δ_{nm} is Kronecker's delta.

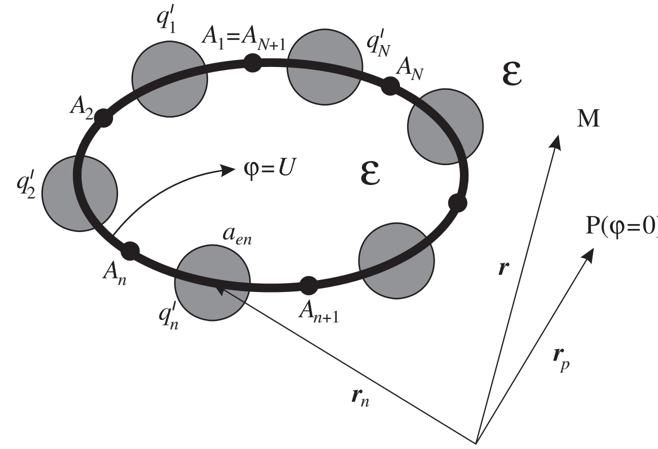

Fig. 1. Cross-section of single infinite extremely thin electrode.

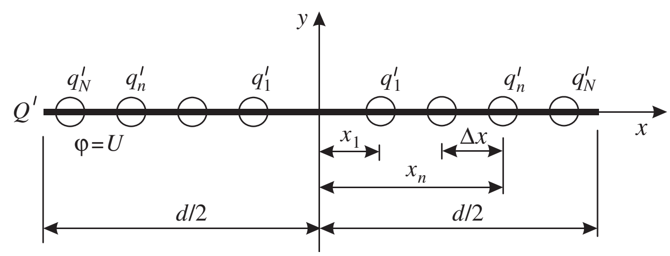

Fig. 2. Flat strip line having approximately zero width.

Table 1
Iterative procedure convergence in the case of strip line, for $N=50$ used equivalent electrodes.

Serial number of iteration	Proportionality factor k	
0	0.0,500,000,000,000,000	0.5,000,000,000,000,000
1	0.2,444,814,734,136,710	0.2,543,064,344,116,882
2	0.2,507,438,012,117,270	0.2,509,186,209,791,079
3	0.2,508,558,473,918,113	0.2,508,589,438,215,607
4	0.2,508,578,321,609,972	0.2,508,578,870,008,971
5	0.2,508,578,673,126,848	0.2,508,578,682,839,356
6	0.2,508,578,679,352,445	0.2,508,578,679,524,460
7	0.2,508,578,679,462,705	0.2,508,578,679,465,751
8	0.2,508,578,679,464,655	0.2,508,578,679,464,711
9	0.2,508,578,679,464,691	0.2,508,578,679,464,691
10	0.2,508,578,679,464,691	0.2,508,578,679,464,691

After solving this system of linear equations, ER of strip line is

$$A_e=kd=\exp\frac{\sum_{n=1}^{N}q'_n\,\log\sqrt{(x_1+x_n)^2[(x_1-x_n)^2+k^2(\Delta x)^2\delta_{1n}]}}{2\sum_{n=1}^{N}q'_n}, \qquad (4)$$

which presents a nonlinear equation, after whose solving the proportionality factor k is obtained.

Specifically, the obtained nonlinear equation can be approximately numerically solved using the iterative procedure, assuming the initial solution for k and putting it on the right side of Eq. (4). The new, more accurate value of this factor is obtained on the left side. The given exact value can be achieved after several iterations, as it can be seen from Table 1.

The obtained results are much more accurate if the number of EEs is greater, as can be seen from Table 2.

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