Engineering Analysis with Boundary Elements 35 (2011) 729–734

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Boundary knot method for heat conduction in nonlinear functionally graded material

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ARTICLE INFO

Article history:
Received 13 August 2010
Accepted 11 December 2010
Available online 26 January 2011

Keywords:
Boundary knot method
Kirchhoff transformation
Nonlinear functionally graded material
Heat conduction
Meshless

ABSTRACT

This paper firstly derives the nonsingular general solution of heat conduction in nonlinear functionally graded materials (FGMs), and then presents boundary knot method (BKM) in conjunction with Kirchhoff transformation and various variable transformations in the solution of nonlinear FGM problems. The proposed BKM is mathematically simple, easy-to-program, meshless, high accurate and integration-free, and avoids the controversial fictitious boundary in the method of fundamental solution (MFS). Numerical experiments demonstrate the efficiency and accuracy of the present scheme in the solution of heat conduction in two different types of nonlinear FGMs.

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1. Introduction

Functionally graded materials (FGMs) are a new generation of composite materials whose microstructure varies from one material to another with a specific gradient. In particular, "a smooth transition region between a pure ceramic and pure metal would result in a material that combines the desirable high temperature properties and thermal resistance of a ceramic, with the fracture toughness of a metal" [1]. In virtue of their excellent behaviors, FGMs have become more and more popular in material engineering and have featured in a wide range of engineering applications (e.g., thermal barrier materials [2], optical materials [3], electronic materials [4] and ever biomaterials [5])

During the past decades extensive studies have been carried out on developing numerical methods for analyzing the thermal behavior of FGMs, for example, the finite element method (FEM) [6], the boundary element method (BEM) [7,8], the meshless local boundary integral equation method (LBIE) [9], the meshless local Petrov–Galerkin method (MLPG) [10–13] and the method of fundamental solution (MFS) [14–16]. However, the conventional FEM is inefficient for handling materials whose physical property varies continuously; BEM needs to treat the singular or hyper-singular integrals [17,18], which is mathematically complex and requires additional computing costs. It is worth noting that, with the exception of mesh-based FEM and BEM, the other above-mentioned methods are classified to the meshless method. Among these meshless methods, LBIE and MLPG belong to the category of weak-formulation, and MFS belongs to the category of strong-formulation.

This study focuses on strong-formulation meshless methods due to their inherent merits on easy-to-program and integration-free. The MFS distributes the boundary knots on fictitious boundary [19] outside the physical domain to avoid the singularities of fundamental solutions, and selecting the appropriate fictitious boundary plays a vital role for the accuracy and reliability of the MFS solution, however, it is still arbitrary and tricky task, largely based on experiences.

Later, Chen and Tanaka [20] develops an improved method, boundary knot method (BKM), which used the nonsingular general solution instead of the singular fundamental solution and thus circumvents the controversial artificial boundary in the MFS. This study first derives the nonsingular general solution of heat conduction in FGM, and then applies the BKM in conjunction with the Kirchhoff transformation to heat transfer problems with nonlinear thermal conductivity. A brief outline of the paper is as follows: Section 2 describes the BKM coupled with Kirchhoff transformation for heat conduction in nonlinear FGM, followed by Section 3 to present and discuss the numerical efficiency and accuracy of the proposed approach in two typical examples. Finally some conclusions are summarized in Section 4.

2. Boundary knot method for nonlinear functionally graded material

Consider a heat conduction problem in an anisotropic heterogeneous nonlinear FGM, occupying a 2D arbitrary shaped region

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doi:10.1016/j.enganabound.2010.11.013

$\Omega \subset \Re^2$ bounded by its boundary Γ, and in the absence of heat sources. Its governing differential equation is stated as

$$\sum_{i,j=1}^{2} \frac{\partial}{\partial x_i}\left(K_{ij}(x,T)\frac{\partial T(x)}{\partial x_j}\right)=0, \quad x \in \Omega \tag{1}$$

with the following boundary conditions.

Dirichlet/essential condition:

$$T(x)=\overline{T}, \quad x \in \Gamma_D \tag{2a}$$

Neumann/natural condition:

$$q(x)=-\sum_{i,j=1}^{2} K_{ij}\frac{\partial T(x)}{\partial x_j}n_i(x)=\overline{q}, \quad x \in \Gamma_N \tag{2b}$$

Robin/convective condition:

$$q(x)=h_e(T(x)-T_\infty), \quad x \in \Gamma_R \tag{2c}$$

where T is the temperature, $\Gamma=\Gamma_D+\Gamma_N+\Gamma_R$ and $K=\{K_{ij}(x,T)\}_{1 \le i,j \le 2}$ denotes the thermal conductivity matrix which satisfies the symmetry ($K_{12}=K_{21}$) and positive-definite ($\Delta_K=det(K)=K_{11}K_{22}-K_{12}^2>0$) conditions. $\{n_i\}$ the outward unit normal vector at boundary $x \in \partial\Omega$,h_e the heat conduction coefficient and T_∞ the environmental temperature.

In this study, we assume the heat conductor is an exponentially functionally graded material such that its thermal conductivity can be expressed by

$$K_{ij}(x,T)=a(T)\overline{K}_{ij}e^{\sum_{i=1}^{2}2\beta_i x_i}, \quad x=(x_1,x_2) \in \Omega \tag{3}$$

in which $a(T)>0$,$\overline{K}=\{\overline{K}_{ij}\}_{1 \le i,j \le 2}$ is a symmetric positive definite matrix, and the values are all real constants. β_1 and β_2 denote constants of material property characteristics.

By employing the Kirchhoff transformation

$$\phi(T)=\int a(T)\,dT \tag{4}$$

Eqs. (1) and (2) can be reduced as the following form:

$$\left(\sum_{i,j=1}^{2}\left(\overline{K}_{ij}\frac{\partial^2 \Phi_T(x)}{\partial x_i \partial x_j}+2\beta_i\overline{K}_{ij}\frac{\partial \Phi_T(x)}{\partial x_j}\right)\right)e^{\sum_{i=1}^{2}2\beta_i x_i}=0, \quad x \in \Omega \tag{5}$$

$$\Phi_T(x)=\phi(\overline{T}), \quad x \in \Gamma_D \tag{6a}$$

$$q(x)=-\sum_{i,j=1}^{2}K_{ij}\frac{\partial T(x)}{\partial x_j}n_i(x)=-e^{\sum_{i=1}^{2}2\beta_i x_i}\sum_{i,j=1}^{2}\overline{K}_{ij}\frac{\partial \Phi_T(x)}{\partial x_j}n_i(x)=\overline{q}, \; x \in \Gamma_N \tag{6b}$$

$$q(x)=h_e\left(\Phi_T(x)-\varphi(T_\infty)\right), \quad x \in \Gamma_R \tag{6c}$$

where $\Phi_T(x)=\varphi(T(x))$and the inverse Kirchhoff transformation

$$T(x)=\varphi^{-1}(\Phi_T(x)) \tag{7}$$

And then we derive the nonsingular general solution of Eq. (5) by two-step variable transformations:

Step 1: To simplify the expression of Eqs. (5), let $\Phi_T=\Psi e^{-\sum_{i=1}^{2}\beta_i(x_i+s_i)}$. Eq. (5) can then be rewritten as follows:

$$\left(\sum_{i,j=1}^{2}\overline{K}_{ij}\frac{\partial \Psi(x)}{\partial x_i \partial x_j}-\lambda^2\Psi(x)\right)e^{\sum_{i=1}^{2}\beta_i(x_i+s_i)}=0, \quad x \in \Omega \tag{8}$$

where $\lambda=\sqrt{\sum_{i=1}^{2}\sum_{j=1}^{2}\beta_i\overline{K}_{ij}\beta_j}$. Since $e^{\sum_{i=1}^{2}\beta_i(x_i+s_i)}>0$. The Trefftz functions of Eq. (8) are equal to those of anisotropic modified Helmholtz equation.

Step 2: To transform the anisotropic Eq. (8) into isotropic one, we set

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}=\begin{pmatrix} 1/\sqrt{\overline{K}_{11}} & 0 \\ -\overline{K}_{12}/\sqrt{\overline{K}_{11}\Delta_{\overline{K}}} & \sqrt{\overline{K}_{11}}/\sqrt{\Delta_{\overline{K}}} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{9}$$

where $\Delta_{\overline{K}}=\det(\overline{K})=\overline{K}_{11}\overline{K}_{22}-\overline{K}_{12}^2>0$.

It follows from Eq. (8) that

$$\left(\sum_{i=1}^{2}\frac{\partial^2\Psi(y)}{\partial y_i \partial y_i}-\lambda^2\Psi(y)\right)=0, \quad y \in \Omega \tag{10}$$

Therefore, Eq. (10) is the isotropic modified Helmholtz equation, the corresponding nonsingular solution can be found in [20]. Then the nonsingular solution of Eq. (8) can be obtained by using inverse transformation (9),

$$u_G(x,s)=-\frac{1}{2\pi\sqrt{\Delta_{\overline{K}}}}I_0(\lambda R) \tag{11}$$

in which $R=\sqrt{\sum_{i=1}^{2}\sum_{j=1}^{2}r_i\overline{K}_{ij}^{-1}r_j}$,$r_1=x_1-s_1$,$r_2=x_2-s_2$, where-$x$,$s$are collocation points and source points, respectively, and I_0 denotes the zero-order modified Bessel function of first kind.

Finally, by implementing the variable transformation $\Phi_T=\Psi e^{-\sum_{i=1}^{2}\beta_i(x_i+s_i)}$, the nonsingular solution of Eq. (5) is in the following form:

$$u_G(x,s)=-\frac{I_0(\lambda R)}{2\pi\sqrt{\Delta_{\overline{K}}}}e^{-\sum_{i=1}^{2}\beta_i(x_i+s_i)} \tag{12}$$

It is worth noting that the source points are placed on the physical boundary by using the present nonsingular general solution u_G.

In the boundary knot method, the solution of Eqs. (5) and (6) is approximated by a linear combination of general solutions with the unknown expansion coefficients as shown below:

$$\overline{\Phi}(x)=\sum_{i=1}^{N}\alpha_i u_G(x,s_i) \tag{13}$$

where $\{\alpha_i\}$ are the unknown coefficients determined by boundary conditions. After $\Phi(x)$ is obtained, the temperature solution T to Eqs. (1) and (2) can be obtained using Eq. (7).

The heat flux can then be given by

$$q(x)=\sum_{i=1}^{N}\alpha_i Q(x,s_i) \tag{14}$$

in which

$$\begin{aligned} Q(x,s_i)&=\sum_{i,j=1}^{2}\overline{K}_{ij}\frac{\partial u_G(x,s_i)}{\partial x_j}n_i(x)e^{\sum_{i=1}^{2}2\beta_i x_i} \\ &=\frac{e^{\sum_{i=1}^{2}2\beta_i r_i}}{2\pi\sqrt{\Delta_{\overline{K}}}}\left(-\frac{\lambda}{R}I_1(\lambda R)\sum_{i=1}^{2}n_i(x)r_i+I_0(\lambda R)\sum_{i=1}^{2}\sum_{j=1}^{2}n_i(x)\overline{K}_{ij}\beta_j\right) \end{aligned} \tag{15}$$

where I_1 denotes the first-order modified Bessel function of first kind.

In view of the general solution satisfying the governing Eq. (5), a priori, the presented method only needs boundary discretization to satisfy boundary conditions

$$A\alpha=b \tag{16}$$

in which

$$A=\begin{pmatrix} u_G(x_j,s_i) \\ Q(x_j,s_i) \\ Q(x_j,s_i)-h_e u(x_j,s_i) \end{pmatrix} \tag{17a}$$

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