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Analysis of functionally graded plates by meshless method: A purely analytical formulation

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ABSTRACT

Functionally graded plates under static and dynamic loads are investigated by the local integral equation method (LIEM) in this paper. Plate bending problem is described by the Reissner moderate thick plate theory. The governing equations for the functionally graded material with respect to the neutral plane are presented in the Laplace transform domain and therefore the in-plane and bending problems are uncoupled. Both isotropic and orthotropic material properties are considered. The local integral equation method is developed with the locally supported radial basis function (RBF) interpolation. As the closed forms of the local boundary integrals are obtained, there are no domain or boundary integrals to be calculated numerically in this approach. The solutions of the nodal values for the entire plate are obtained by solving a set of linear algebraic equation system with certain boundary conditions. Details of numerical procedures are presented and the accuracy and convergence characteristics of the method are examined. Several examples are presented for the functionally graded plates under static and dynamic loads and the accuracy for proposed method has been observed compared with 3D analytical solutions.

1. Introduction

The boundary element method (BEM) is an effective numerical tool for the solution of boundary or initial-boundary value problems [1]. The first application of the boundary integral equation method to Reissner's plate theory was given by Van der Ween [2]. Dynamic analysis of elastic Reissner plates has been performed by the direct BEM in the frequency domain [3,4]. Unfortunately, the number of applications to analyse functionally graded plates by the use of BEM is very limited due to the lack of availability of fundamental solutions.

Meshfree methods have achieved remarkable progress in recent years. Typical methods include the method of fundamental solution [5,6], the diffuse element method [7], the element-free Galerkin method [8] and the meshless local Petrov–Galerkin method [9]. These methods have attracted much attention during the past decade, especially owing to their high adaptivity and low effort to prepare input data for numerical analyses [10]. The meshless collocation method, meshless local Petrov–Galerkin method and local boundary integral equation method (LBIE) with moving least square and radial basis function interpolations has been developed by Ferreira et al. [11], Gilhooley et al. [12], Sladek et al. [13] and Li et al. [14], respectively, for anisotropic non-homogeneous and composite plates. Recently an improved meshless collocation method has been proposed for two elastostatic and elastodynamic problems by Wen and Aliabadi [15] and for nonlinear analysis by Wen and Hon [16]. A comprehensive review of meshless methods can be found by Atluri [17] and Liu [18].

The interpolation methods including the radial basis function method and moving least squares method play an important role in the meshless investigations. Hardy [19] developed a general scattered data approximation method, called multi-quadric (MQ), to approximate two-dimensional geographical surfaces. Numerical results show that the MQ method offers a very accurate interpolation. In Franke's review [20] paper, MQ was rated one of the best methods among 29 scattered data interpolation schemes, based on accuracy, stability, efficiency, memory requirement and ease of implementation. Kansa [21] derived a modified MQ scheme for solving PDE problems. In the last two decades, the developments of radial basis functions as a truly meshless method has drawn the attention of many investigators (see Golberg and Chen [22]).

Functionally graded materials (FGMs) are multi-phase materials with the phase volume fractions varying gradually in space, in a pre-determined profile. This results in continuously graded mechanical properties that vary gradually with location within the material. Generally, the spatial gradients in material behaviour render FGMs as superior to conventional composites. FGMs

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differ from composites wherein the volume fraction of the inclusion is uniform throughout the composite. The closest analogous of FGMs are laminated composites, but the latter possess distinct interfaces. More details about FGMs can be referred to references [23,24]. The most familiar FGMs are compositionally graded from a refractory ceramic to a metal. It can incorporate incompatible functions such as the heat, wear and corrosion resistances of ceramics and the high toughness, high strength and bonding capability of metals without severe internal thermal stresses. In this paper, the meshless local integral equation method is presented for the FGMs plate with Reissner's plate theory. With the use of radial basis functions, the analytical solutions for all domain integrals in the weak form are derived. The weak formulations for the governing equations with a unit test function are obtained exactly for the local domain integrals. As the closed form of the local domain integrals are obtained, the computational time is reduced significantly. The Laplace transform technique is applied for dynamic problems. Numerical results for isotropic and orthotropic plates with various boundary conditions and static and Heaviside loadings are presented to illustrate the applicability of the proposed method.

2. Governing equations for FGM plates

Consider a FGMs rectangular plate of uniform thickness h, length a and width b as shown in Fig. 1. The Cartesian coordinate system is introduced such that the bottom and top surfaces of plate is placed in the plane $z'=0$ and $z'=h$. Then, the spatial displacement field has the following form [1,2]:

$$u=u_0+(z'-z_0)w_1(x,y,t)$$
$$v=v_0+(z'-z_0)w_2(x,y,t)$$
$$w=w_3(x,y,t) \tag{1}$$

where z_0 indicates the position of neutral plane where the in-plane stresses are zero, u_0 and v_0 displacements of in-plane, w_1, w_2 are rotations around x and y axes, respectively, and w_3 is the out-of-plane deflection. The linear strains are given by

$$\varepsilon_x=u_{0,1}+(z'-z_0)w_{1,1}$$
$$\varepsilon_y=v_{0,2}+(z'-z_0)w_{2,2}$$
$$\gamma_{xy}=u_{0,2}+v_{0,1}+(z'-z_0)(w_{1,2}+w_{2,1})$$
$$\gamma_{xz}=w_1+w_{3,1}$$
$$\gamma_{yz}=w_2+w_{3,2} \tag{2}$$

For a plane stress problem in orthotropic materials, the Hook's law can be written in matrix form as

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{bmatrix}=\mathbf{D}(x,y,z)\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix} \tag{3}$$

where material parameter matrix

$$\mathbf{D}(x,y,z')=\begin{bmatrix}E_1/e & E_1v_{21}/e & 0 & 0 & 0\\ E_2v_{12}/e & E_2/e & 0 & 0 & 0\\ 0 & 0 & G_{12} & 0 & 0\\ 0 & 0 & 0 & G_{13} & 0\\ 0 & 0 & 0 & 0 & G_{23}\end{bmatrix} \tag{4}$$

in which $e=1-v_{12}v_{21}$, E_1 and E_2 are the Young's moduli referring to the axes x and y, respectively. G_{12}, G_{13} and G_{23} are shear moduli and v_{ij} are Poisson's ratios. There are two main assumptions for the functionally graded materials. The first assumption is that the material properties are graded along the plate thickness as

$$P(x,y,z')=P_b(x,y)+[P_t(x,y)-P_b(x,y)](z'/h)^n \tag{5}$$

and the section assumption is

$$P(x,y,z')=P_b(x,y)\exp(z'\delta/h),\quad \delta=\ln(P_t/P_b) \tag{6}$$

where P_t and P_b denote the properties of the top and bottom faces of the plate, respectively, and n is a parameter that indictates the material variation profile. In addition, Possion's ratios and mass density of the plate are assumed to be constants. Under pure bending, we have

$$\int_0^h \sigma_x(x,y,z')dz'=0 \tag{7}$$

Therefore, the position of the neutral plane is obtained for the first assumption of material variation profile:

$$z_0=\frac{(n+1)(n\beta+2)}{2(n+2)(n\beta+1)}h. \tag{8}$$

For the second assumption of variation profile, one holds

$$z_0=\left(\frac{1}{1-\beta}+\frac{1}{\ln\beta}\right)h \tag{9}$$

where $\beta=P_b(x,y)/P_t(x,y)$. It should be noticed that the neutral plane does not exist on the middle plane ($z'=h/2$) of the plate except $\beta=1$.

New coordinate system is established on the neutral plane as shown in Fig. 1, i.e. $z=z'-z_0$, and the bending moments $M_{\alpha\beta}$ and shear forces Q_α are defined as [1]

$$\begin{bmatrix}M_{11}\\ M_{22}\\ M_{12}\end{bmatrix}=\int_{-z_0}^{h-z_0}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}z\,dz,\quad \begin{bmatrix}Q_1\\ Q_2\end{bmatrix}=\kappa\int_{-z_0}^{h-z_0}\begin{bmatrix}\tau_{13}\\ \tau_{23}\end{bmatrix}dz$$

$$\text{and}\quad \begin{bmatrix}N_{11}\\ N_{22}\\ N_{12}\end{bmatrix}=\int_{-z_0}^{h-z_0}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}dz \tag{10}$$

where $\kappa=5/6$ in the Reissner plate theory. Substituting (2) and (3) into moment and force resultants (10) yields

$$M_{11}=D_{11}w_{1,1}+D_{12}w_{2,2}$$
$$M_{22}=D_{21}w_{1,1}+D_{22}w_{2,2}$$
$$M_{12}=S(w_{1,2}+w_{2,1})$$

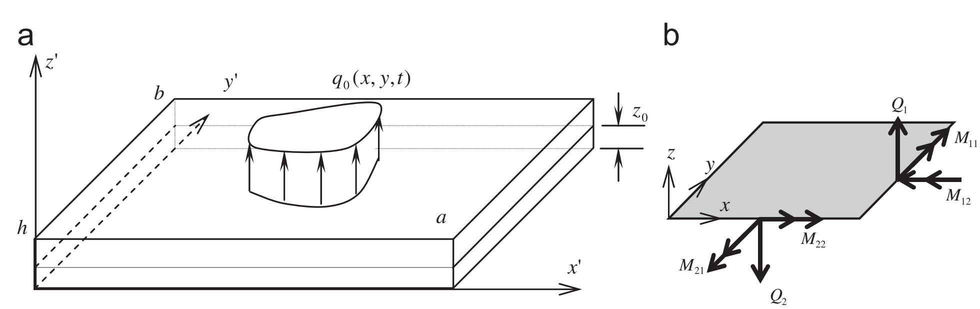

Fig. 1. A rectangular plate and coordinate system and sign convention of bending moments and shear forces for the plate on neutral plane.