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Analysis of isotropic and laminated plates by an affine space decomposition for asymmetric radial basis functions collocation

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ABSTRACT

As a first endeavor, this paper applies an affine space decomposition, proposed by Ling and Hon, to the static analysis of laminated plates. The radial basis functions collocation method by Kansa is modified by this affine space decomposition. The present approach can be seen as an improvement of the original Kansa's method, producing better conditioned matrices and very stable solutions for the static analysis of laminated plates. A static analysis of isotropic and laminated plates is performed by considering a first-order shear deformation plate theory. The equilibrium equations and the boundary conditions are interpolated by collocation with radial basis functions.

1. Introduction

This paper addresses for the first time the analysis of isotropic and laminated composite plates by radial basis functions collocation and an affine space decomposition, by Ling and Hon [1].

Some relevant works on analysis of thick plates include those of Wang et al. [6], Khdeir and Librescu [7], Bhimaraddi [8], Kitipornchai et al. [9], Liew et al. [10–12,40], Putcha and Reddy [13], and Reddy and Phan [14]. An historical review on laminated plates and shells has been presented by Carrera [15]. The use of alternative methods to the finite element methods such as meshless methods is attractive due to the absence of a mesh. A recent review of meshless methods including the element-free Galerkin method and reproducing kernel particle method can be found in [41]. Another mesh-free method, the radial point interpolation method, is referenced as being accurate when dealing with scattered nodes in the domain [42,43].

The focus of this paper is the radial basis functions (RBF) collocation method, which have been previously studied by numerous authors for the analysis of structures and materials [16–27]. More recently the authors have applied RBFs to the static deformations of composite beams and plates [28–30].

Although much work has been done with analytical or meshless methods, there is no research on static analysis of isotropic and laminated plates by radial basis functions collocation with an affine space decomposition.

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2. Solution of partial differential equations (PDE) problems with radial basis functions

The radial basis function (ϕ) approximation of a function ($\mathbf{u}$) is given by

$$\tilde{\mathbf{u}}(x)=\sum_{i=1}^{N}\alpha_i\phi(\|x-y_i\|_2),\quad x\in\mathbb{R}^n \tag{1}$$

where y_i, $i=1,\ldots,N$ is a finite set of distinct points (centers) in $\mathbb{R}^n$. The coefficients α_i are chosen so that $\tilde{\mathbf{u}}$ satisfies some boundary conditions. In this work are considered the following RBFs:

$$\phi(r)=\sqrt{1+\epsilon^2r^2}\quad \text{multiquadrics}$$

$$\phi(r)=(\epsilon+r^2)^{-1}\quad \text{inverse quadrics}$$

where the Euclidian distance r is real and non-negative, and ϵ is a (positive) shape parameter.

2.1. Solution of the interpolation problem

Hardy [31] introduced multiquadrics in the analysis of scattered geographical data. In the 1990s Kansa [32] used multiquadrics for the solution of partial differential equations.

Considering N distinct interpolations points, and given $u(x_j)$, $j=1,2,\ldots,N$, the vector of the coefficients α_i is the solution of a $N\times N$ linear system

$$\mathbf{A}\underline{\alpha}=\mathbf{u} \tag{2}$$

where $\mathbf{A}=[\phi(\|x_j-y_i\|_2)]_{N\times N}$, $\underline{\alpha}=[\alpha_1,\alpha_2,\ldots,\alpha_N]^T$ and $\mathbf{u}=[u(x_1), u(x_2),\ldots,u(x_N)]^T$. The RBF interpolation matrix A is positive definite for some RBFs [33], but in general provides ill-conditioned systems.

2.2. Solution of the static problem

The solution of a static problem by radial basis functions considers N_I nodes in the domain and N_B nodes on the boundary, with total number of nodes $N = N_I + N_B$.

Denoting the sampling points by $x_i \in \Omega$, $i = 1, \ldots, N_I$ and $x_i \in \partial\Omega$, $i = N_I + 1, \ldots, N$. At the points in the domain it is solved the following system of equations:

$$\sum_{i=1}^{N} N\alpha_i \mathcal{L}\phi(\|x_j - y_i\|_2) = \mathbf{f}(x_j), \quad j = 1, 2, \ldots, N_I \tag{3}$$

or

$$\mathcal{L}^I \underline{\alpha} = \mathbf{F} \tag{4}$$

The system of equations for the boundary conditions is

$$\sum_{i=1}^{N} \alpha_i \mathcal{L}_B \phi(\|x_j - y_i\|_2) = \mathbf{g}(x_j), \quad j = N_I + 1, \ldots, N \tag{5}$$

or

$$\mathbf{B}\underline{\alpha} = \mathbf{G} \tag{6}$$

Therefore the finite-dimensional static problem can be written as

$$\begin{bmatrix} \mathcal{L}^I \\ \mathbf{B} \end{bmatrix} \underline{\alpha} = \begin{bmatrix} \mathbf{F} \\ \mathbf{G} \end{bmatrix} \tag{7}$$

where

$$\mathcal{L}^I = [\mathcal{L}\phi(\|x_j - y_i\|_2)]_{N_I \times N}, \quad \mathbf{B} = [\mathcal{L}_B \phi(\|x_j - y_i\|_2)]_{N_B \times N}$$

By inverting the system (7), it is obtained the vector of coefficients $\underline{\alpha}$. Next step is the calculus of the solution by the interpolation equation (1).

3. Affine approach

For certain values of the shape parameter in the RBF-PDE problem, the system of equations can be ill-conditioned [1,45,46]. To overcome this problem, an improved solution based on an affine space decomposition method that decouples the influence between the interior and boundary collocations was proposed by Ling and Hon [1]. A short introduction to the procedure is given below.

Concerning to system (7), if $\mathbf{B}^\dagger$ is the pseudoinverse matrix and $\boldsymbol{\psi}_\mathbf{B}$ is the null matrix of $\mathbf{B}$ obtained from the SVD (singular value decomposition) of $\mathbf{B}$, then the coefficients vector $\underline{\alpha}$ may be written as

$$\underline{\alpha} = \mathbf{B}^\dagger \mathbf{G} + \boldsymbol{\psi}_\mathbf{B} \underline{\beta} \tag{8}$$

where

$$\mathbf{B}\boldsymbol{\psi}_\mathbf{B} = 0 \tag{9}$$

and

$$\mathbf{B}\mathbf{B}^\dagger \mathbf{G} = \mathbf{G} \tag{10}$$

Substituting the coefficient vector $\underline{\alpha}$ in the part of the resultant system (7) that corresponds to the operator $\mathcal{L}$ leads to a reduced system with $\underline{\beta}$ being the new coefficient vector

$$[\mathcal{L}^I \;\; \boldsymbol{\psi}_\mathbf{B}]\underline{\beta} = [\mathbf{F} - \mathcal{L}^I \;\; \mathbf{B}^\dagger \;\; \mathbf{G}] \tag{11}$$

Finally the solution $\underline{\alpha}$ is recovered from Eq. (8). This is an equivalent formulation of the problem with the benefit of a generally better conditioned system matrix.

This affine space approach is now generalized and applied to systems of PDEs. Consider the following boundary value problem with two PDEs, each one of them decoupled into interior portion and boundary portion:

$$\begin{cases} \mathcal{L}_1 u(x) = \mathbf{f}_1(x), & x \in \Omega \\ \mathcal{L}_{B1} u(x) = \mathbf{g}_1(x), & x \in \partial\Omega \\ \mathcal{L}_2 u(x) = \mathbf{f}_2(x), & x \in \Omega \\ \mathcal{L}_{B2} u(x) = \mathbf{g}_2(x), & x \in \partial\Omega \end{cases} \tag{12}$$

The rows of the resultant system of the RBF collocation method are rearranged so that the boundary conditions appear together. The rearranged resultant system in matrix form is given by

$$\begin{bmatrix} \mathcal{L}_1^I \\ \mathcal{L}_2^I \\ \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \underline{\alpha} = \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix} \tag{13}$$

The coefficient vector $\underline{\alpha}$ is now decomposed by the orthonormal basis of the null space of

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \tag{14}$$

Next the pseudoinverse matrix $\mathbf{B}^\dagger$ and the null matrix $\boldsymbol{\psi}_\mathbf{B}$ are obtained from the SVD of $\mathbf{B}$. As a result of the affine decomposition it follows a reduced system

$$[\mathcal{L}^I \;\; \boldsymbol{\psi}_\mathbf{B}]\underline{\beta} = [\mathbf{F} - \mathcal{L}^I \;\; \mathbf{B}^\dagger \;\; \mathbf{G}] \tag{15}$$

where

$$\mathcal{L}^I = \begin{bmatrix} \mathcal{L}_1^I \\ \mathcal{L}_2^I \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix} \tag{16}$$

After solving the reduced system using again SVD, its solution vector β is employed to obtain the vector of coefficients $\underline{\alpha}$ through the relation

$$\underline{\alpha} = \mathbf{B}^\dagger \mathbf{G} + \boldsymbol{\psi}_\mathbf{B} \underline{\beta} \tag{17}$$

4. Equations of equilibrium for the plate in bending

Based on the FSDT (first-order shear deformation theory), the transverse displacement $w(x,y)$ and the rotations $\theta_x(x,y)$ and $\theta_y(x,y)$ about the y- and x-axes are independently interpolated due to uncoupling between inplane displacements and bending displacements for plates. For static analysis are considered the following equations of equilibrium, corresponding to the so-called Mindlin–Reissner theory for plates [2–4]:

$$D_{11}\frac{\partial^2\theta_x}{\partial x^2} + D_{16}\frac{\partial^2\theta_y}{\partial x^2} + (D_{12}+D_{66})\frac{\partial^2\theta_y}{\partial x\partial y} + 2D_{16}\frac{\partial^2\theta_x}{\partial x\partial y} + D_{66}\frac{\partial^2\theta_x}{\partial y^2} + D_{26}\frac{\partial^2\theta_y}{\partial y^2} - kA_{45}\left(\theta_y + \frac{\partial w}{\partial y}\right) - kA_{55}\left(\theta_x + \frac{\partial w}{\partial x}\right) = 0 \tag{18}$$

$$D_{16}\frac{\partial^2\theta_x}{\partial x^2} + D_{66}\frac{\partial^2\theta_y}{\partial x^2} + (D_{12}+D_{66})\frac{\partial^2\theta_x}{\partial x\partial y} + 2D_{26}\frac{\partial^2\theta_y}{\partial x\partial y} + D_{26}\frac{\partial^2\theta_x}{\partial y^2} + D_{22}\frac{\partial^2\theta_y}{\partial y^2} - kA_{44}\left(\theta_y + \frac{\partial w}{\partial y}\right) - kA_{45}\left(\theta_x + \frac{\partial w}{\partial x}\right) = 0 \tag{19}$$

$$\frac{\partial}{\partial x}\left[kA_{45}\left(\theta_y + \frac{\partial w}{\partial y}\right) + kA_{55}\left(\theta_x + \frac{\partial w}{\partial x}\right)\right]$$