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Dynamic fracture analysis of a smoothly inhomogeneous plane containing defects by BEM

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In this work, the dynamic interaction between defects of different types such as cracks and cavities in a smoothly inhomogeneous, elastic anisotropic plane subjected to incident SH-waves is investigated.

Solution of the ensuing boundary-value problem is numerically realized using the non-hypersingular, traction boundary element method (BEM). By employing a special functional transform, the wave equation for inhomogeneous media is reduced to one with constant coefficients and the relevant frequency-dependent fundamental solution for graded anisotropic continua is obtained by the Radon transform. All surface discretizations are then done by standard collocation procedure with a parabolic type of approximation of all field variables. Next, validation of the numerical method is carried out through comparisons with available solutions for crack stress intensity factors (SIFs) and for cavity stress concentration factors (SCF).

A detailed parametric study is then undertaken for a circular cavity interacting with a stationary, mode III crack in the presence of a propagating SH-wave. In sum, the key parameters of the simulation study are the characteristics of the incident wave, the geometry and configuration of the defects, the material inhomogeneity, and the dynamic interaction between the defects. The influence of all these key parameters on the dynamic SIF and SCF for different defects is finally discussed.

1. Introduction

In recent years, considerable effort has been devoted to the subject of interaction of elastic waves with different types of defects in a solid. This basic problem is the underlying topic in: (a) computational fracture mechanics, where near-field solutions for stress concentrations are important, see Pao and Mow [1], Lu and Hanyga [2], Dong and Lee [3], Ayatollahi et al. [4], Meguid and Wang [5], Hirose [6], Wang et al. [7], Hasebe et al. [8], Zhang and Gross [9], Dineva et al. [10] and (b) evaluation of material strength, fatigue and service life of structures and structural components, see Kita et al. [11], He et al. [12], Park et al. [13], Chaudhuri and Chakraborty [14], Haukaas and Kiureghian [15], Tekie and Ellingwood [16].

The interaction between defects such as cavities and cracks is important, because the stress concentration around cavity boundaries may generate cracks in their vicinity, see Gong and Meguid [17], Liaw and Kamel [18], Chen and Wang [19], Liu and Liu [20]. In geophysics, it is necessary also to account for the dynamic interaction between cavities and cracks in soil deposits and to evaluate the influence of this phenomenon on the elastic wave motion generated at the ground surface. It can be seen from a survey of the literature that the results for dynamic interaction between different types of defects are mostly for homogeneous materials. For instance, the static problem for an in-plane crack emanating from a cavity in an elastic isotropic infinite plane is considered in Wang et al. [7] and in Hasebe et al. [8], while the same problem but in an elastic anisotropic plane is discussed in Dong and Lee [3]. Dynamic in-plane wave scattering problems for a single cavity of elliptic, circular and arbitrary form are considered in Hirose [6] and in Pao and Mow [1]. Furthermore, the anti-plane elastodynamic problem for a crack–cavity system in a homogeneous elastic isotropic infinite solid was solved by Lu and Hanyga [2], Meguid and Wang [5] and Ayatollahi et al. [4]. A hypersingular integral equation method based on Green's function for a point harmonic force applied on the infinite plane with a circular cavity was used by Lu and Hanyga [2], while the distributed dislocation technique was employed by Ayatollahi et al. [4] to carry out stress analysis in planes with cracks and cavities subjected to a time-harmonic anti-plane excitation. Finally, the dynamic interaction of a crack with a circular hole has been solved by a singular integral equation method using

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Chebyshev polynomials at different frequencies of the anti-plane load by Meguid and Wang [5].

There are relatively few results on wave scattering by a crack or an inclusion in a smoothly inhomogeneous media with position-dependent material parameters, even in the absence of multiple defects and their interaction. Such types of solutions were obtained by Manolis [21], who studied elastic transient wave scattering around an in-plane cavity in the infinite inhomogeneous plane by a displacement-type BEM, by Manolis et al. [22] and Dineva et al. [23], who investigated near and far wave fields in an infinite inhomogeneous plane with a finite line crack under time-harmonic P- and SV-incident waves using a non-hypersingular, traction-based BEM and by Zhang et al. [24], who presented a transient anti-plane crack analysis in the inhomogeneous infinite plane by a hyper-singular, traction-type time domain BEM. Wave scattering phenomena due to a subsurface crack in an inhomogeneous half-plane and its influence on ground surface seismograms was studied by the BEM in Dineva [25]. Finally, the non-hypersingular BEM was implemented by Daros [26] for modeling a single anti-plane crack in an infinite anisotropic inhomogeneous plane, and numerical results were presented in the form of SIF for three different types of material inhomogeneities, namely the exponential, sinusoidal and quadratic in terms of the depth coordinate.

From a structural engineering viewpoint, the construction of large underground structures such as nuclear power plants, oil reservoirs and tunnels necessitates a detailed dynamic analysis plus an evaluation of the influence on the surrounding media from the point of view of earthquake-resistant design. In much of the literature, the underground structure–ground system is considered as the inclusion-half space model, see Selberg [27], Gamer [28], Datta and El-Akily [29], Gregory [30], Doyum and Erodogan [31], Shah et al. [32]. This makes the problem similar, from a mechanics point of view, with what has been discussed in the above.

In sum, the following conclusions can be drawn based on evaluation of the state of the art in the field:

(a) Much of the work done to date is for the static case and for homogeneous isotropic solids.
(b) The most commonly used analytical and semi-analytical methods for stress concentration calculations are the wave function expansion, the matched asymptotic expansion, integral transforms and singular integral equation methods. However, the application of analytical methods for analysis of smoothly inhomogeneous solids with defects suffers from a number of drawbacks. The most important is the lack of generality in the solution process, since only a very restricted class of problems can be treated regarding the geometry of the defects and their configuration.
(c) There is a paucity of results for dynamic problems on crack–cavity formations in graded anisotropic media, capable of taking into consideration the sensitivity of both stress intensity factors at the crack tips and stress concentration factors on the rim of the cavity to incoming wave frequency content and propagation direction, the material inhomogeneity, the geometry and mutual disposition of the defects and their interaction.

To the authors' knowledge, a BEM solution for the time-harmonic response of an inhomogeneous elastic anisotropic plane with multiple defects has not been presented in the literature so far. Thus, the main aim of this paper is to develop accurate and efficient non-hypersingular traction based BEM for solving the steady-state, anti-plane wave propagation problem in a smoothly inhomogeneous elastic, anisotropic plane with defects of different types and accounting for their interaction.

The paper is organized as follows: first, the problem statement is given in Section 2. Next, the non-hypersingular traction-based BEM is discussed in Section 3 and its numerical implementation is outlined in Section 4. This is followed by a detailed validation study of the proposed numerical scheme, followed by a series of simulations and their interpretation. Finally, Section 5 presents some concluding remarks.

2. Statement of the problem

Consider an infinitely extending, linearly elastic inhomogeneous and anisotropic solid in a Cartesian coordinate system $Ox_1x_2x_3$ as shown in Fig. 1. The solid is swept by an incident time-harmonic SH-wave, polarized along the x_3 axis and propagating in the plane $x_3=0$. The wave has a frequency content ω and an incident angle θ with respect to the Ox_1 axis. Due to the time-harmonic behaviour of all field variables, the common multiplier $e^{i\omega t}$ is suppressed in the following. The deformation of the solid is anti-plane strain, and the only non-zero quantities are displacement component $u_3(x,\omega)$ and shear stress components $\sigma_{i3}(x,\omega)$, $i=1$, 2, where $x=(x_1,x_2)$. The solid contains multiple defects in the form of M finite, stationary mode III cracks Γ^{cr}_m, $m=1,2,\ldots,M$ with a half-length c_m and N circular cavities H_k, $k=1,2,\ldots,N$ of radius R_k and center C_k, $\partial H_k=\Gamma^h_k$, as shown in Fig. 1. It is possible to consider cavities of arbitrary shape, but in what follows we restrict ourself to circular cylindrical ones as being representative of geotechnical tunnel construction. We will, however, include the case of an elliptical cavity in the validation study to demonstrate the generality of our BEM formulation. Next denote $\Gamma^{cr}=\bigcup_{m=1}^{M}\Gamma^{cr}_m$, $\Gamma^h=\bigcup_{k=1}^{N}\Gamma^h_k$, $\Gamma=\Gamma^{cr}\bigcup\Gamma^h$ and suppose $\bigcap_{m=1}^{M}\Gamma^{cr}_m=\emptyset$, $\bigcap_{k=1}^{N}\Gamma^h_k=\emptyset$, $\Gamma^{cr}\bigcap\Gamma^h=\emptyset$, i.e. the defects do not intersect each other. The angle between the coordinate axis Ox_1 and the principle axis of the material symmetry Ox'_1 is denoted by χ. The case of general anisotropy is considered, i.e. $\chi\neq 0$ or $\chi\neq\pi/2$ and three material parameters c_{44},c_{45},c_{55} are necessary to characterize the matrix of the elastic coefficients such that

$$c_{44}>0,\quad c_{55}>0,\quad c_{44}c_{55}-c_{45}^2>0. \tag{1}$$

In the case of orthotropic anisotropy, the condition $c_{45}=0$ is satisfied. If the solid is transversely isotropic and the axis of material symmetry is along the Ox_3 axis the conditions $c_{45}=0$ and $c_{44}=c_{55}$ are satisfied, i.e. the plane $x_3=0$ is isotropic.

Wave propagation in anisotropic elastic inhomogeneous solids is governed by the wave equation, which in the absence of body

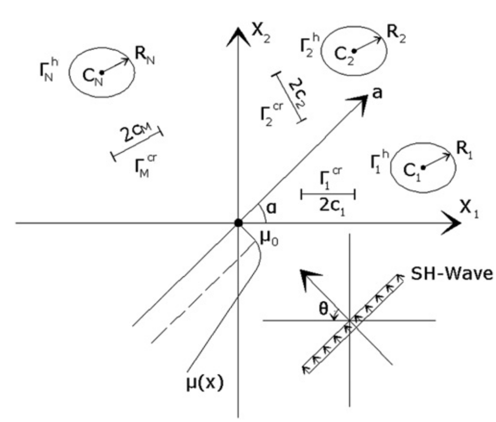

Fig. 1. Inhomogeneous infinite anisotropic solid with multiple defects (mode III cracks and circular cavities) under a SH-wave propagating in the $x_3=0$ plane with incident angle θ. The material gradient has magnitude $|a|$ and direction α.