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Hybrid extended displacement discontinuity-charge simulation method for analysis of cracks in 2D piezoelectric media

MingHao Zhao *, GuangTao Xu, CuiYing Fan

Department of Engineering Mechanics, Zhengzhou University, No. 100 Science Road, Zhengzhou, Henan Province 450001, PR China

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ABSTRACT

The extended displacement discontinuity method (EDDM) and the charge simulation method (CSM) are combined to develop an efficient approach for analysis of cracks in two-dimensional piezoelectric media. In the proposed hybrid EDD–CSM, the solution for an electrically impermeable crack is approximately expressed by a linear combination of fundamental solutions of the governing equations, which includes the extended point force fundamental solutions with the sources placed at chosen points outside the domain of the problem under consideration and the extended Crouch fundamental solutions with the extended displacement discontinuities placed on the crack. The coefficients of the fundamental solutions are determined by letting the approximated solution satisfy the conditions on the boundary of the domain and on the crack face. Furthermore, the hybrid EDD–CSM is applied to solve the problems of cracks under electrically permeable condition, as well as under semi-permeable conditions by using an iterative approach. Two important crack problems in fracture mechanics, the center cracks and the edge cracks in piezoelectric strips, are analyzed by the proposed method. The stress intensity factor and the electric displacement intensity factor are calculated. Meanwhile the effects of strip size and the electric boundary conditions on these intensity factors are studied.

1. Introduction

Because of the coupling effect between mechanical and electric properties, more and more applications are being found on the piezoelectric ceramics for smart structures and systems. Due to their brittleness, the research on fracture of piezoelectric materials has been attracting many attentions. The Stroh formalism, the potential function method, and the boundary integral equation method have been used for analysis of cracks in piezoelectric media. Reviews of this research topic can be found in Refs. [1–4]. As we know, it is difficult to obtain an analytical solution of a generally practical problem with a finite domain. Numerical method, such as the finite element method (FEM) or the boundary element method (BEM), is an alternatively effective way, which has been studied intensively and extensively (e.g., Refs. [5,6]). BEM is one of the most efficient methods to the problems of stress concentrations and singularities. Several techniques have been developed to overcome the ill-conditioned problem in fracture mechanics of piezoelectric material: the special Green's function method [6], the dual BEM [7], and the extended displacement discontinuity method (EDDM) [8,9]. This issue has been well documented in BEM of conventional elastic media [10,11].

The electric field in the cavity of a crack makes the fracture problem of piezoelectric materials even more complicated. Two kinds of electric boundary conditions, known as the electrically impermeable crack and electrically permeable crack, are widely used on crack faces. In fact, they are only two extreme cases of a real crack. The appropriateness of these approximate electric boundary conditions was clarified in Refs. [2,12]. Indeed the crack opens under applied mechanical–electric loadings, while the electric field in the crack cavity greatly depends on the crack opening and the permeability of the material in the crack cavity. In this regard, the crack opening model [13] that considers the electric field in the crack cavity and is sometimes called the semi-permeable crack [2] is thought to be most appropriate to the facts. It is a typical geometrically nonlinear problem. The EDDM grasps the basic characteristic of a crack across which the extended displacement is discontinuous, therefore, above-mentioned each kind of electric boundary conditions can be easily incorporated in the EDDM and the implementation is very natural. It has been proven that the EDDM is much more effective in crack analysis of piezoelectric media [14–17].

On the other hand, the charge simulation method (CSM) is also a boundary method similar to the method of fundamental solution (MFS) [18] and the virtual boundary element method (VBEM) [19]. In the CSM, the solution is approximated by a linear

* Corresponding author. Tel.: +86 371 6778 1752; fax: +86 371 6778 1237.
E-mail addresses: memhzhao@zzu.edu.cn, memhzhao@sina.com (M. Zhao).

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combination of fundamental solutions of the governing equations in terms of the sources placed at chosen fixed points outside the domain of the problem under consideration. The coefficients of the fundamental solutions are determined by letting the approximated solution satisfy the boundary conditions at corresponding points on the boundary of the domain of the problem. The CSM shares all the advantages of the BEM over domain discretization methods. Furthermore, it does not require discretization of the boundary, and, thus, avoiding integrations over the boundary. No quadrature is required to evaluate the solution field in the interior of the domain. The data preparation is little. This method has been successfully applied to solutions of plane elastic problems [20–22], deflection and vibration of plates [23–25], axisymmetric problems in elastostatics [26], and three-dimensional (3D) problems in elastostatics [27].

In this paper, the above two efficient methods, i.e., the EDDM and the CSM, are combined to develop an approach of higher computing speed and efficiency in crack analysis in two-dimensional (2D) finite piezoelectric media. Following this Introduction, the basic equations are outlined in Section 2; the hybrid Extended Displacement Discontinuity (EDD)–CSM is presented in Section 3; the proposed hybrid EDD–CSM is employed in Section 4 to analyze the center crack and the edge crack in piezoelectric strips; and finally, the present paper is concluded in Section 5.

2. Basic equation

Consider a plane problem of a transversely isotropic piezoelectric medium, which occupies a finite domain Ω bounded by Γ, as shown in Fig. 1. The Cartesian coordinate system oxz is set up such that the polarization direction of the piezoelectric medium is along the z-axis. There is a line crack S parallel to the x-axis in the piezoelectric medium, with the upper and lower crack faces being denoted by S^+ and S^-, respectively.

The extended equilibrium equations, in the absence of body force and free electric charge, are given by [1–3]

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}=0,\quad \frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \sigma_z}{\partial z}=0,\quad \frac{\partial D_x}{\partial x}+\frac{\partial D_z}{\partial z}=0, \tag{1}$$

where σ_x, σ_z and τ_{xz} denote the stress components; D_x and D_z are the electric displacements in the x- and z-direction, respectively.

The extended strain–displacement relations are expressed as

$$\varepsilon_x=\frac{\partial u}{\partial x},\quad \varepsilon_z=\frac{\partial w}{\partial z},\quad \gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x},\quad E_x=-\frac{\partial \phi}{\partial x},\quad E_z=-\frac{\partial \phi}{\partial z}, \tag{2}$$

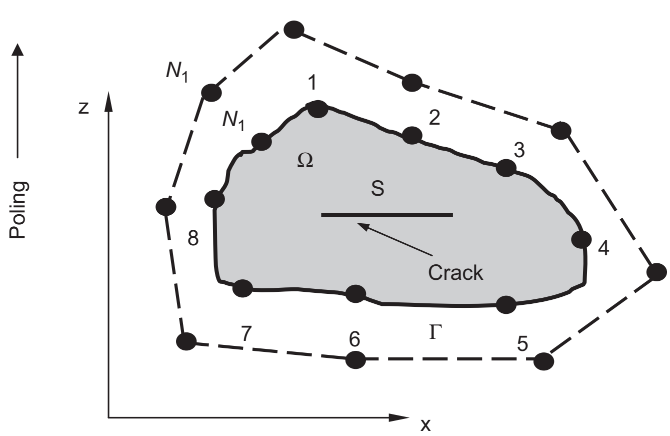

Fig. 1. Source points and collocation points on boundary of a transversely isotropic piezoelectric medium.

where ε_x, ε_z and γ_{xz} are the strain components; E_x and E_z are electric fields in the x- and z-directions, respectively; u and w are elastic displacement components, and ϕ the electric potential.

The constitutive equations read as

$$\begin{aligned}
\sigma_x &= c_{11}\varepsilon_x + c_{13}\varepsilon_z - e_{31}E_z,\\
\sigma_z &= c_{13}\varepsilon_x + c_{33}\varepsilon_z - e_{33}E_z,\\
\tau_{xz} &= c_{44}\gamma_{xz} - e_{15}E_x,\\
D_x &= e_{15}\gamma_{xz} + \varepsilon_{11}E_x,\\
D_z &= e_{31}\varepsilon_x + e_{33}\varepsilon_z + \varepsilon_{33}E_z,
\end{aligned} \tag{3}$$

where c_{11}, c_{13}, c_{33} and c_{44} denote the elastic constants, e_{31}, e_{33} and e_{15} the piezoelectric constants, and ε_{11} and ε_{33} the dielectric permittivities.

There are two categories of boundary conditions on boundary Γ. One is the mechanical condition and the other is the electric condition. The mechanical condition can be divided into the prescribed displacement boundary condition

$$u=\bar{u},\quad w=\bar{w},\quad \text{on } \Gamma_u, \tag{4a}$$

where the over bar "–" denotes the prescribed values on the boundary, and the prescribed traction boundary condition

$$t_x \equiv \sigma_x n_x + \tau_{xz} n_z = \bar{t}_x,\quad t_z \equiv \tau_{xz} n_x + \sigma_z n_z = \bar{t}_z,\quad \text{on } \Gamma_t, \tag{4b}$$

where $\{n_i\}$ is the unit vector normal to boundary Γ and outward from the domain.

Similarly, the electric boundary condition can be divided into the prescribed potential boundary condition

$$\phi=\bar{\phi},\quad \text{on } \Gamma_\phi, \tag{5a}$$

and the prescribed electric displacement boundary condition

$$\omega \equiv D_x n_x + D_z n_z = \bar{\omega},\quad \text{on } \Gamma_\omega. \tag{5b}$$

The mechanical boundary condition on crack face S has the same form as that given in Eq. (4). And the electric boundary condition takes one of the following three kinds of electric boundary conditions on crack faces [2], i.e.,

$$D_z^+ = D_z^- = -\bar{\omega}, \tag{6a}$$

for electrically impermeable condition, where the superscripts "+" and "−" denote the quantities on the upper and lower crack faces, respectively, with the unit outward normal vector being $\{n_i^+\} = -\{n_i^-\} = \{0, -1\}$,

$$D_z^+ - D^c = D_z^- - D^c = -\bar{\omega},\quad \phi^+ = \phi^-, \tag{6b}$$

for electrically permeable condition, where D^c denotes the electric displacement along the z-axis in the crack cavity, and

$$D_z^+ - D^c = D_z^- - D^c = -\bar{\omega},\quad D^c = -\varepsilon^c[\phi^+ - \phi^-]/[w^+ - w^-], \tag{6c}$$

for electrically semi-permeable condition, where ε^c is the dielectric constant of the material in the crack cavity.

3. Hybrid EDD–CSM

Based on the CSM [20–22], N_1 collocation points are chosen on the boundary Γ, and correspondingly N_1 source points are taken outside the domain Ω, as schematically shown in Fig. 1. Unknown extended concentrated load $P_{ki}(k=1,2,\ldots,N_1;\ i=1,2,3)$ is applied at source point k, where P_{k1} and P_{k2} are the mechanical loads, respectively, along the x- and z-axis, and P_{k3} is the point electric charge. The crack S is discretized into N_2 elements, and unknown EDD $\|u_{kj}\|(\equiv u_{kj}^+ - u_{kj}^-)$ $(k=1,2,\ldots,N_2;\ j=1,2,3)$ is uniformly distributed on each element, where $\|\cdot\|$ denotes the discontinuity of an extended displacement across the crack. By using the extended point force fundamental solutions given in Appendix A, the extended Crouch fundamental solutions given in