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Simulation of groundwater flow in unconfined aquifer using meshfree point collocation method

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ABSTRACT

For appropriate management of available groundwater, the flow behavior in the porous media has to be analyzed. The complex problem of groundwater flow can be studied by solving the governing equations analytically or by using numerical methods. As the analytical solutions are available only for simple idealized cases, numerical methods such as finite difference method (FDM) and finite element method (FEM) are generally used for field problems. Meshfree (MFree) method is an alternative numerical approach to solve complex groundwater problems in simple manner. MFree method eliminates the drawback of meshing and remeshing as in FDM and FEM which can translate to substantial cost and time savings in modeling. In this paper, a model using MFree point collocation method (PCM) with multi-quadric radial basis function (MQ-RBF) is proposed for 2D groundwater flow simulation. The accuracy of the developed model is verified with available analytical solution in literature. The developed model is applied initially for a hypothetical problem and further for a field problem to compute head distribution. The PCM model results for the hypothetical problem are compared with FEM simulations while that of field problem are compared with boundary element based model results. The PCM model results are found to be satisfactory showing the applicability of the present approach.

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1. Introduction

Groundwater is one of the components of the hydrological cycle through which humanity gets the supply of water for domestic, industrial and agriculture uses. The complex problem of groundwater flow can be studied by solving the governing equations of groundwater flow analytically or numerically. The analytical solutions are available only for the most simplifying conditions where the velocities are constant, hydraulic conductivity remains constant and source terms are simple functions. For most of the field problems, numerical models are to be used based on numerical techniques such as FDM and FEM [16]. But while using FDM or FEM, a grid or mesh has to be formed over the domain and tedious pre-processing has to be carried out.

Meshfree (MFree) method [11] is an alternative numerical approach to solve complex engineering problems by simple and accurate manner. This method is used to transform the governing partial differential equations into a system of algebraic equations for the whole problem domain without the use of pre-defined mesh. MFree method uses a set of nodes scattered within the problem domain as well as on the boundaries of the domain to represent the problem domain and its boundaries.

Since last decade, researchers and engineers are trying to solve groundwater problems with MFree methods. Kansa [7] used multiquadrics (MQ) as the spatial approximation scheme for parabolic, hyperbolic and the elliptic Poisson's equation and shown that MQ is an accurate approximation scheme for interpolation and partial derivative estimates for a variety of two-dimensional functions over both gridded and scattered data. Lin and Atluri [10] proposed meshless Petrov–Galerkin (MLPG) method to solve steady convection diffusion problems, in 1D and 2D and found that even for very high Peclet number flows, the model gives very good results. Atluri and Shen [1] showed that the MLPG method is less expensive, both in computational costs as well as human-labor costs, compared to FEM, or boundary element method (BEM). Li et al. [9] developed a meshless method using collocation method with radial basis functions for modeling groundwater contaminant transport and concluded that the method is simple, easily applicable and accurate. Liu [12] applied radial point collocation method (RPCM) to solve convection diffusion 2D Burgers equation with the interpolation schemes in locally supported domains based on radial basis functions and observed that the method is attractive in solving computational fluid problems. Dehghan and Shokri [5] proposed a numerical scheme to solve the 2D time-dependent Schrödinger equation using collocation points and approximating the solution using multi-quadrics (MQ) and thin plate splines (TPS)

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based radial basis function (RBF) and found that the results of numerical experiments showed good agreement with analytical solutions. Kee et al. [8] developed a stabilized MFree method based on the strong formulation and local approximation using RBF and also established a residual based error indicator and showed that the features of the MFree strong-form method can facilitate an easier implementation of adaptive analysis. Praveen Kumar and Dodagoudar [14] developed a numerical model based on radial point interpolation method (RPIM) for 2D contaminant transport through saturated porous media and showed that the proposed model is in good agreement with the experimental results.

In this paper, a meshfree model is developed for the simulation of groundwater flow in unconfined aquifer. The model is based on collocation techniques with multi-quadric radial basis function (MQ-RBF). The developed model is verified with analytical solution and further applied to hypothetical and field problems and compared with FEM and BEM solutions, respectively, and found to be satisfactory.

2. Governing equations and boundary conditions

The governing equation describing the flow in an unconfined aquifer in two dimensions is given as [3]

$$\frac{\partial}{\partial x}\left(K_x h\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y h\frac{\partial h}{\partial y}\right)=S_y\frac{\partial h}{\partial t}\pm Q_w\delta(x-x_i)(y-y_i) \tag{1}$$

The following initial conditions are used

$$h(x,y,0)=h_0(x,y) \quad x,y\in\Omega \tag{2}$$

For the unconfined aquifer problems, the boundary conditions used are:

$$h(x,y,t)=h_1(x,y,t) \quad x,y\in\partial\Omega_1$$

$$Kh\frac{\partial h}{\partial n}=q_2(x,y,t) \quad x,y\in\partial\Omega_2 \tag{3}$$

where $h(x,y,t)$ is the piezometric head (m); K is the hydraulic conductivity (m/d); K_x, K_y are the hydraulic conductivities in x and y directions; S_y is the specific yield; x, y are the horizontal space variables (m); Q_w is the source or sink function ($-Q_w$=Source, $+Q_w$=Sink) (m^3/d/m^2); t is the time in days; δ is the Dirac delta function with the property that when $x=x_i$ and $y=y_i$, $\delta=1$ but$=0$ elsewhere; Ω is the flow region; $\partial\Omega$ is the boundary region ($\partial\Omega_1\cup\partial\Omega_2=\partial\Omega$); $\partial/\partial n$ is the normal derivative; $h_0(x,y)$ is the initial head in flow domain (m); $h_1(x,y,t)$ is the known head value of the boundary head (m); $q_2(x,y,t)$ is the known inflow rate (m^3/d/m).

3. Numerical formulation

In this study, a meshfree formulation based on point collocation method (PCM) with MQ-RBF is used to develop the flow model. The PCM formulation for flow model is given in the following section.

For transient groundwater flow in unconfined aquifer in 2D without pumping or recharge, Eq. (1) can be written as [3]

$$\frac{\partial}{\partial x}\left(K_x h\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y h\frac{\partial h}{\partial y}\right)=S_y\frac{\partial h}{\partial t} \tag{4}$$

By considering the aquifer to be homogeneous and isotropic, Eq. (4) can be written as

$$K\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)+K\frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y}\right)=S_y\frac{\partial h}{\partial t} \tag{5}$$

If K is constant, differentiating by parts gives

$$K\left[\left(\frac{\partial h}{\partial x}\right)\left(\frac{\partial h}{\partial x}\right)+h\frac{\partial^2 h}{\partial x^2}\right]+K\left[\left(\frac{\partial h}{\partial y}\right)\left(\frac{\partial h}{\partial y}\right)+h\frac{\partial^2 h}{\partial y^2}\right]=S_y\frac{\partial h}{\partial t} \tag{6}$$

In PCM, the first step is to define the trial solution $\hat{h}(x,y,t)$ as [11]

$$\hat{h}(x,y,t)=\sum_{i=1}^{n}h_i(t)R_i(x,y) \tag{7}$$

where n is the number of nodes considered and $R_i(x,y)$ is the shape function.

Here, shape function can be written as [11]

$$R_i(x,y)=\sqrt{(x-x_i)^2+(y-y_i)^2+Cs^2} \tag{8}$$

where, x, y are the co-ordinates of the point of interest in the support domain; x_i, y_i are the co-ordinates of ith node in the support domain; $Cs=\alpha_c d_c$. Here, α_c is the shape parameter and d_c is the nodal spacing in the support domain. Its first and second derivative with respect to x and y can be written as

$$\frac{\partial R_i(x,y)}{\partial x}=\frac{(x-x_i)}{R_i(x,y)};\quad \frac{\partial R_i(x,y)}{\partial y}=\frac{(y-y_i)}{R_i(x,y)}, \tag{9a, b}$$

$$\frac{\partial^2 R_i(x,y)}{\partial x^2}=\frac{((y-y_i)^2+Cs^2)}{{R_i}^3(x,y)};\quad \frac{\partial^2 R_i(x,y)}{\partial y^2}=\frac{(x-x_i)^2+Cs^2}{{R_i}^3(x,y)} \tag{10a, b}$$

For time discretization, fully implicit finite forward difference approximation is used as [13,16]

$$\frac{\partial h}{\partial t}=\langle R_i(x,y)\rangle\left(\frac{1}{\Delta t}\left(\left\{h_i^{t+\Delta t}\right\}-\left\{h_i^t\right\}\right)\right) \tag{11}$$

where $\left\{h_i^{t+\Delta t}\right\}$ and $\{h_i^t\}$are the nodal heads at the time $(t+\Delta t)$ and t, respectively.

Therefore, using Eqs. (7), (8), (9a,b) and (10a,b), Eq. (6) can be written as

$$\begin{aligned}&K\left[\left(\frac{\partial R_i(x,y)}{\partial x}\right)h_i^t\left(\frac{\partial R_i(x,y)}{\partial x}\right)h_i^{t+\Delta t}+(R_i(x,y)h_i^t)\left(\frac{\partial^2 R_i(x,y)}{\partial x^2}\right)h_i^{t+\Delta t}\right]\\&\quad+K\left[\left(\frac{\partial R_i(x,y)}{\partial y}\right)h_i^t\left(\frac{\partial R_i(x,y)}{\partial y}\right)h_i^{t+\Delta t}+(R_i(x,y)h_i^t)\left(\frac{\partial^2 R_i(x,y)}{\partial y^2}\right)h_i^{t+\Delta t}\right]\\&\quad=\frac{S_y}{\Delta t}\left(R_i(x,y)\left(h_i^{t+\Delta t}-h_i^t\right)\right)\end{aligned} \tag{12}$$

Here, $R_i(x,y)$, $(\partial R_i(x,y)/\partial x)$, $(\partial R_i(x,y)/\partial y)$, $(\partial^2 R_i(x,y)/\partial x^2)$ and $(\partial^2 R_i(x,y)/\partial y^2)$ values are to be calculated for each support domain and then they are incorporated in the global matrix for whole problem domain. Further Eq. (12) can be written in the matrix form as

$$\begin{pmatrix}\frac{K\Delta t}{S_y}\left([K_2]\{h_i^t\}[K_2]\left\{h_i^{t+\Delta t}\right\}+[K_1]\{h_i^t\}[K_3]\left\{h_i^{t+\Delta t}\right\}\right)+\\ \frac{K\Delta t}{S_y}\left([K_4]\{h_i^t\}[K_4]\left\{h_i^{t+\Delta t}\right\}+[K_1]\{h_i^t\}[K_5]\left\{{h_i}^{t+\Delta t}\right\}\right)\end{pmatrix}=\left([K_1]\left\{h_i^{t+\Delta t}-h_i^t\right\}\right) \tag{13}$$

By rearranging the terms in Eq. (13), we get

$$\left([K_1]-\begin{pmatrix}\frac{K\Delta t}{S_y}([K_2]\{h_i^t\}[K_2]+[K_1]\{h_i^t\}[K_3])+\\ \frac{K\Delta t}{S_y}([K_4]\{h_i^t\}[K_4]+[K_1]\{h_i^t\}[K_5])\end{pmatrix}\right)\left\{{h_i}^{t+\Delta t}\right\}=([K_1]\{h_i^t\}) \tag{14}$$

where $[K_1]$ is the global matrix of shape function, $[K_2]$ is the global matrix of first derivative of shape functions with respect to x, $[K_3]$ is the global matrix of second derivative of shape functions with respect to x, $[K_4]$ is the global matrix of first derivative of shape functions with respect to y, $[K_5]$ is the global matrix of second derivative of shape functions with respect to y. For incorporating the boundary conditions, the nodes lying on the boundaries should be assigned the properties of boundary values depending on

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