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Natural convection of micropolar fluid in an enclosure with boundary element method

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ABSTRACT

The contribution deals with numerical simulation of natural convection in micropolar fluids, describing flow of suspensions with rigid and underformable particles with own rotation. The micropolar fluid flow theory is incorporated into the framework of a velocity–vorticity formulation of Navier–Stokes equations. The governing equations are derived in differential and integral form, resulting from the application of a boundary element method (BEM). In integral transformations, the diffusion-convection fundamental solution for flow kinetics, including vorticity transport, heat transport and microrotation transport, is implemented. The natural convection test case is the benchmark case of natural convection in a square cavity, and computations are performed for Rayleigh number values up to 10^7. The results show, which microrotation of particles in suspension in general decreases overall heat transfer from the heated wall and should not therefore be neglected when computing heat and fluid flow of micropolar fluids.

1. Introduction

Micropolar fluids are a subclass of microfluids, introduced by Eringen [1]. A simple microfluid is by Eringen's definition a fluid medium whose properties and behaviour are influenced by the local motions of the material particles contained in each of its volume elements. A microfluid is an isotropic viscous fluid and possesses local inertia. Because of a complex formulation for a general microfluid this class of fluids is divided into subclasses, which allow a simplified description of the effects arising from particle micromotions. As mentioned, in micropolar fluids, a subclass of microfluids, rigid particles contained in a small volume element can rotate about the center of the volume element, which is described by the microrotation vector [2,3]. This local rotation of the particles is independent of the mean fluid flow and its local vorticity field. Lukaszewicz [4] presented in his book mathematical aspects of the micropolar fluid flow theory. From this theory it is also expected to successfully describe non-Newtonian behaviour of certain fluids, such as liquid crystals, ferro liquids, colloidal fluids, liquids with polymer additives, animal blood carrying deformable particles (platelets), clouds with smoke, suspensions, slurries and liquid crystals.

In the recent years this theory is gaining interest of a lot of researchers. One of the reasons is progress in a micromachining technology and in the opinion of a few scientists which say that flows on the microscale differ from that on a macroscale, described by the Navier–Stokes equations. Papautsky et al. [5] described microchannel fluid flow behaviour with numerical model based on a micropolar fluid flow theory and experimentally verified the model. Results showed that micropolar fluid flow theory present better agreement with an experiment, than with the use of classical Navier–Stokes theory. Applicability of the theory of the micropolar fluids in a microchannel also depends on the geometrical dimension of the flow field [6].

Natural convection is a physical phenomenon, where due to the presence of a temperature difference between body surfaces buoyancy forces appeared. Most fluids near a hot wall will have their density decreased, and an upward near wall motion will be induced. Natural convection of a micropolar fluid in a rectangular enclosure was presented in the work of Hsu and Chen [7], where they presented a parametric study of the effect of microstructure on the flow and heat transfer. In their study they use a cubic spline collocation method. They showed that a heat transfer rate and therefore also Nusselt number of a micropolar fluid is decreased compared to the Newtonian fluid. In another work Hsu et al. [8] investigated natural convection of micropolar fluids in an enclosure with a single or multiple uniform heat sources, where also different boundary conditions for microrotation were considered. In this work, the heat transfer characteristics and flow phenomenon are presented for different values of the Rayleigh number, tilting angle of the enclosure and various material properties of the micropolar fluids. The results indicate that

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dependence of a microrotation term and heat transfer on a microstructure parameter is significant. A similar work was also published by Aydin and Pop [9], where they presented flow of different Rayleigh and Prandtl numbers. In all studies the results are compared with benchmark results of a Newtonian fluid from Davies [10].

Among different approximation methods for solving problems of fluid flow the boundary element method (BEM) is a relatively new method with some interesting features, described in Refs. [11–13]. Here, we will focus on the development of BEM for velocity–vorticity formulation of Navier–Stokes equations presented by Škerget et al. [11] and show how to incorporate the micropolar fluid theory into the BEM framework. In the paper von Škerget et al. [12] also presented result of natural convection with the use of Navier–Stokes equations and BEM approximation method.

2. Mathematical formulation

For description of compressible viscous fluid flow we use conservation laws for mass, momentum and energy with appropriate rheological models and equations of state. In the case of fluids, which behaviour can be described by micropolar fluid flow theory Eringen [2] presented modified equation of conservation laws for mass (1), momentum (2), microrotation (4) and in the case of natural convection also conservation of energy (3)

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}(\rho\vec{v})=0 \tag{1}$$

$$\rho\frac{D\vec{v}}{Dt}=-\vec{\nabla}p+(\lambda_v+2\mu_v+k_v)\vec{\nabla}\vec{\nabla}\cdot\vec{v}-(\mu_v+k_v)\vec{\nabla}\times\vec{\nabla}+k_v\vec{\nabla}\times\vec{N}+\rho\vec{f} \tag{2}$$

$$c_p\rho\frac{DT}{Dt}=\lambda\Delta T \tag{3}$$

$$\rho j\frac{D\vec{N}}{Dt}=(\alpha_v+\beta_v+\gamma_v)\vec{\nabla}\vec{\nabla}\cdot\vec{N}-\gamma_v\vec{\nabla}\times\vec{\nabla}\times\vec{N}+k_v\vec{\nabla}\times\vec{v}-2k_v\vec{N}+\rho\vec{l} \tag{4}$$

Differential operator $D(\cdot)/Dt=\partial(\cdot)/\partial t+v_k\partial(\cdot)/\partial x_k$ represents the Stokes material derivative. In the next step we assume that fluid mass density ρ, specific isobaric heat per unit mass c_p, heat conduction λ and all micropolar fluid properties as second-order viscosity coefficient λ_v, dynamic viscosity μ_v, vortex viscosity coefficient k_v, viscosity gradient coefficients α_v, β_v, γ_v and microinertia j, are constant parameters. We also consider zero couples $\vec{l}$ and in case of buoyancy the Boussinesq assumption is used. Therefore we can rewrite Eqs. (1)–(4) for the assumption that micropolar fluid flow will be viscous, incompressible, steady state and laminar

$$\vec{\nabla}\cdot\vec{v}=0 \tag{5}$$

$$\rho\frac{D\vec{v}}{Dt}=-\vec{\nabla}p+(2\mu_v+k_v)\vec{\nabla}\vec{\nabla}\cdot\vec{v}-(\mu_v+k_v)\vec{\nabla}\times\vec{\nabla}\times\vec{v}+k_v\vec{\nabla}\times\vec{N}+\vec{g}\beta_T(T-T_0) \tag{6}$$

$$c_p\rho\frac{DT}{Dt}=\lambda\Delta T \tag{7}$$

$$\rho j\frac{D\vec{N}}{Dt}=(\alpha_v+\beta_v+\gamma_v)\vec{\nabla}\vec{\nabla}\cdot\vec{N}-\gamma_v\vec{\nabla}\times\vec{\nabla}\times\vec{N}+k_v\vec{\nabla}\times\vec{v}-2k_v\vec{N} \tag{8}$$

β_T presents thermal expansion factor and T_0 is the representing reference temperature.

Using vector algebra ($\vec{\nabla}\times\vec{\nabla}\times\vec{F}=\vec{\nabla}(\vec{\nabla}\cdot\vec{F})-\Delta\vec{F}$), considering the mass conservation (5) and accounting for the solenoidality of microrotation field the momentum (6) and microrotation (8) conservation equations are further simplified to

$$\rho\frac{D\vec{v}}{Dt}=-\vec{\nabla}p+(\mu_v+k_v)\Delta\vec{v}+k_v\vec{\nabla}\times\vec{N}+\vec{g}\beta_T(T-T_0) \tag{9}$$

$$\rho j\frac{D\vec{N}}{Dt}=\gamma_v\Delta\vec{N}+k_v\vec{\nabla}\times\vec{v}-2k_v\vec{N} \tag{10}$$

To incorporate micropolar fluid flow theory into the framework of velocity–vorticity formulation of Navier–Stokes equations and to apply the BEM approximation method, we must first split the dynamics of the flow into its kinematic and kinetic part. This is done by the use of the derived vector vorticity field function $\vec{\omega}$, obtained as a curl of the compatibility velocity field $\vec{\omega}=\vec{\nabla}\times\vec{v}$, which is a solenoidal vector function by the definition $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{v})=0$. By applying the curl operator to vorticity and using the mass conservation equation (5) for the incompressible fluid flow we get an elliptic Poisson equation for the velocity vector [11]

$$\Delta\vec{v}+\vec{\nabla}\times\vec{\omega}=0 \tag{11}$$

or in tensor notation form

$$\frac{\partial^2 v_i}{\partial x_j\partial x_j}+e_{ijk}\frac{\partial\omega_k}{\partial x_j}=0 \tag{12}$$

Eq. (12) represents the kinematic part of the fluid flow where for a known vorticity field, the corresponding velocity field can be determined.

To compute the kinetic part of the flow we apply the curl operator to the momentum conservation equation (9) and considering that $\vec{\nabla}\cdot\vec{\omega}=0$, $\vec{\nabla}\cdot\vec{v}=0$ and $\vec{\nabla}\cdot\vec{N}=0$ due to the vorticity and microrotation definition and mass conservation equation, it follows:

$$\rho\frac{D\vec{\omega}}{Dt}=(\mu_v+k_v)\Delta\vec{\omega}+(\vec{\omega}\cdot\vec{\nabla})\vec{v}-k_v\vec{\nabla}\vec{N}+\beta_T\vec{\nabla}+(\vec{g}(T-T_0)) \tag{13}$$

This equation shows that the rate of change of the vorticity field is due to viscous diffusion, vortex stretching and twisting, microrotation diffusion and buoyancy force.

For the case of two-dimensional plane flow and accounting for all previous assumptions the final form of equations for kinematic and kinetic part are expressed in Cartesian tensor notation form as

$$\frac{\partial^2 v_i}{\partial x_j\partial x_j}+e_{ij}\frac{\partial\omega}{\partial x_j}=0 \tag{14}$$

$$\rho\frac{D\omega}{Dt}=(\mu_v+k_v)\frac{\partial^2\omega}{\partial x_j\partial x_j}-k_v\frac{\partial^2 N}{\partial x_j\partial x_j}-\beta_T\frac{\partial g_i(T-T_0)}{\partial x_j} \tag{15}$$

$$c_p\rho\frac{DT}{Dt}=\lambda\frac{\partial^2 T}{\partial x_j\partial x_j} \tag{16}$$

$$\rho j\frac{DN}{Dt}=\gamma_v\frac{\partial^2 N}{\partial x_j\partial x_j}+k_v e_{ij}\frac{\partial v_i}{\partial x_j}-2k_v N \tag{17}$$

If we assume that $k_v=0$ the equation for vorticity (15) and microrotation (18) are uncoupled, therefore the flow is independent of the microrotation, and the governing equations now resume the form of the classical Navier–Stokes equations.