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DRBEM solution of natural convection flow of nanofluids with a heat source

S. Gümgüm [a,b,1], M. Tezer-Sezgin [b,c,*]

[a] Department of Mathematics, İzmir University of Economics, 35330 İzmir, Turkey
[b] Institute of Applied Mathematics, Middle East Technical University, 06531 Ankara, Turkey
[c] Department of Mathematics, Middle East Technical University, 06531 Ankara, Turkey

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ABSTRACT

This paper presents the dual reciprocity boundary element method (DRBEM) solution of the unsteady natural convective flow of nanofluids in enclosures with a heat source. The implicit Euler scheme is used for time integration. All the convective terms are evaluated in terms of DRBEM coordinate matrix. The vorticity boundary conditions are obtained from the Taylor series expansion of stream function equation. The results report that the average Nusselt number increases with the increase in both volume fraction and Rayleigh number. It is also observed that an increase in heater length reduces the heat transfer. The average Nusselt number of aluminum oxide-water based nanofluid is found to be smaller than that of copper-water based nanofluid. Results are given in terms of streamlines, isotherms, vorticity contours, velocity profiles and tables containing average Nusselt number for several values of Rayleigh number, heater length, volume fraction, and number of iterations together with CPU times.

1. Introduction

Nanofluids are engineered colloids composed of a base fluid and nanometer sized particles. Conventional heat transfer fluids, such as water, engine oil and ethylene glycol have low heat transfer performance. Therefore, various techniques are applied to enhance the heat transfer of these fluids. One of them is the use of solid particles as an additive suspended into the base fluid. The improved heat transfer performance of nanofluids is due to the fact that the nanoparticles increase the heat capacity, and improve the thermal conductivity of the fluid.

Heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids is investigated for various pertinent parameters by Khanafer et al. [1] using finite-volume approach along with the alternating direction implicit method. They analyze the effect of suspended ultrafine metallic nanoparticles on the fluid flow and heat transfer processes within the enclosure. Stream function-vorticity formulation of the transport equations are solved using finite difference method in [2]. They showed that increasing the buoyancy parameter and volume fraction of nanofluids causes an increase in the average heat transfer coefficient. Tiwari and Das [3] investigated the behaviour of nanofluids inside a two-sided lid-driven differentially heated square cavity using finite volume method. Effect of copper-water nanofluid as a cooling medium has been studied to simulate the behaviour of heat transfer due to laminar natural convection in a differentially heated square cavity in [4] using finite volume approach. They observed that the heat transfer decreases with increase in volume fraction for a particular Rayleigh number. Numerical simulation of natural convection of nanofluids in a square enclosure is studied by Ho et al. [5] using finite volume method. Results demonstrate that the uncertainties associated with different formulas adopted for the effective thermal conductivity, and dynamic viscosity of the nanofluid have a strong bearing on the natural convection heat transfer characteristics in the enclosure. Öztop and Abu-Nada [6] studied heat transfer and fluid flow due to buoyancy forces in a partially heated enclosure using different types of nanoparticles. Finite volume method is used to solve the transport equations. It was found that the heater location effects the flow and temperature fields when using nanofluids. They also studied effects of inclination angle on natural convection in enclosures filled with copper-water nanofluid [7]. Natural convection heat transfer of water-based nanofluids in an inclined enclosure with a heat source is investigated by Öğüt [8]. It is observed that the average heat transfer decreases with an increase in the length of the heater. The governing equations are solved using polynomial differential quadrature method (DQM). Aminossadati and Ghasemi [9] studied natural convection flow with cooling of a localized heat source at the bottom of a nanofluid-filled enclosure. The top and vertical walls of the enclosure are maintained at relatively low temperature. The governing equations are solved with a finite volume method using a SIMPLE algorithm. They also studied [10] natural convection heat transfer in an inclined enclosure filled

* Corresponding author at: Department of Mathematics, Middle East Technical University, 06531 Ankara, Turkey. Tel.: +90 312 2105376; fax: +90 312 2102972.
E-mail addresses: sevin.gumgum@ieu.edu.tr (S. Gümgüm), munt@metu.edu.tr (M. Tezer-Sezgin).
[1] Tel.: +90 232 4888307; fax: +90 232 2792626.

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with a water-CuO nanofluid using the same method. Two opposite walls of the enclosure are insulated and the other two walls are kept at different temperatures. The results indicate that adding nanoparticles into pure water improves its heat transfer performance. In another paper [11] they examine the periodic natural convection in an enclosure filled with nanofluids. A periodic behaviour is found for the flow and temperature fields as a result of the oscillating heat flux.

All of the numerical methods available in the literature which have been applied to simulate natural convection of nanofluids in a square enclosure are domain discretization methods in space coordinates. Thus, the resulting system of algebraic equations is of very large size due to the large number of nodal points in the region, to achieve a good accuracy. The boundary element method is a numerical scheme which discretizes only the boundary of the region reducing the size of the resulting systems. But, a domain integral results in BEM due to the inhomogeneity when the equation is Poisson's type. This causes the lost of boundary only nature of BEM. Then, the DRBEM is made use of to transform this domain integral also to a boundary integral. The DRBEM has also the flexibility of using fundamental solution of Laplace equation, and approximating all the convective terms using coordinate matrix in terms of radial basis functions. The application of DRBEM for solving natural convective flow is given in [12] which uses the coupling of DQM-in time and DRBEM-in space. The results are provided for *Ra* values up to 10^5. DRBEM application is extended to solve natural convection flow of micropolar fluids in [13].

In this paper, the dual reciprocity boundary element method is employed to discretize the spatial derivatives in the stream function-vorticity formulation of the Navier–Stokes and energy transport equations. DRBEM idea is applied to the Laplace operator in each equation by using the fundamental solution of Laplace equation and keeping all the other terms as nonhomogeneity [14]. This makes the computations much easier and less expensive comparing to the BEM since the matrices obtained contain only the fundamental solution and its derivative. The DRBEM reduces all calculations to the evaluation of the boundary integrals only. The resulting matrices contain integrals of logarithmic function or its normal derivative. This fact might be advantageous in geometrically involved situations that are frequently encountered in fluid flow problems. The unknown vorticity boundary conditions are obtained from the Taylor series expansion of stream function equation, and all the convective terms are evaluated by using DRBEM coordinate matrix. DRBEM application of unsteady natural convection flow of nanofluids gives rise to systems of initial value problems in time. The implicit Euler scheme is made use of solving these systems, and all the original unknowns (stream function, vorticity and temperature) are obtained at all transient levels including steady-state.

2. Governing equations

The non-dimensional, unsteady equations of motion and energy for nanofluids can be written in terms of stream function (ψ), vorticity (w) and temperature (T), [9] as follows:

$$\nabla^2\psi = -\omega$$

$$\frac{\mu_{nf}}{\rho_{nf}\alpha_f}\nabla^2\omega = \frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial x} + v\frac{\partial\omega}{\partial y} - Ra\,Pr\,\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}\frac{\partial T}{\partial x}$$

$$\frac{\alpha_{nf}}{\alpha_f}\nabla^2 T = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \qquad (1)$$

where $(x,y)\in\Omega\subset R^2$, $t>0$. *Ra* and *Pr* are the Rayleigh and Prandtl numbers, respectively. The subscripts '*nf*' and '*f*' refer to nanofluid and fluid.

The dimensionless velocities are given as $u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$ and the vorticity is defined by $\omega=\partial v/\partial x - \partial u/\partial y$.

Thermal diffusivity and the effective density of the nanofluid is given by [6]

$$\alpha_{nf} = \frac{k_{eff}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s$$

where k is the thermal conductivity, φ is the nanoparticle volume fraction and C_p is the specific heat at constant pressure. The subscripts '*eff*' and '*s*' refer to effective and solid, respectively. $\alpha_f = k_f/(\rho C_p)_f$ is the thermal diffusivity of the fluid.

The heat capacitance and the thermal expansion coefficient of the nanofluid are defined as [9]

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s$$

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s$$

The viscosity of the nanofluid given by [9]

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}$$

Here μ_f is the dynamic viscosity of the fluid. The effective thermal conductivity of the nanofluid is approximated for the spherical nanoparticles by the Maxwell–Garnetts model [6]

$$\frac{k_{eff}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)}$$

The equations in (1) are supplied with the initial conditions

$$w(x,y,0) = w_0(x,y), \quad T(x,y,0) = T_0(x,y)$$

where $w_0(x,y)$ and $T_0(x,y)$ are given functions of space and time.

Corresponding boundary conditions are shown in Fig. 1. The velocity components are zero on the solid walls. The wall at $x=1$ is cooled and the horizontal walls are adiabatic. The left wall is heated with constant heat flux for a varying length with a parameter ε [8]. The fluid in the enclosure is a water-based nanofluid containing copper (Cu) and aluminum oxide (Al_2O_3) nanoparticles. It is assumed that the base fluid and nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo-physical properties of the nanofluid are assumed to be constant except for the density variation, which is approximated

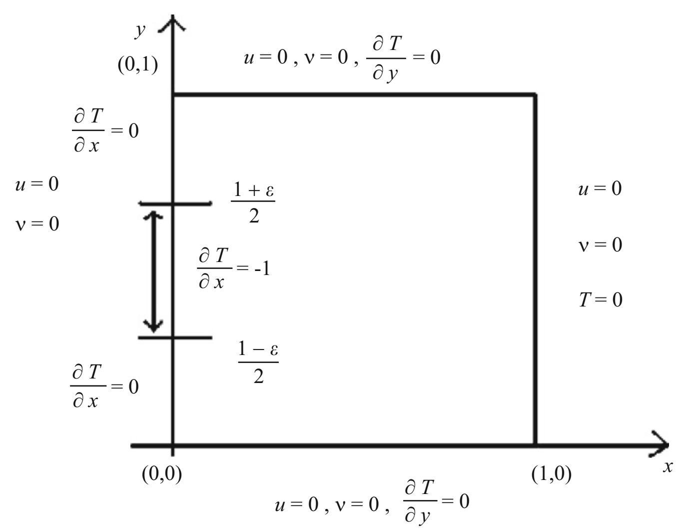

Fig. 1. Layout of the problem.