Engineering Analysis with Boundary Elements 35 (2011) 803–810

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

The multi-domain hybrid boundary node method for 3D elasticity

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ARTICLE INFO

Article history:
Received 10 November 2010
Accepted 11 January 2011
Available online 25 February 2011

Keywords:
Multi-domain
3D elasticity
Hybrid boundary node method
Modified variational principle
Moving Least Squares approximation

ABSTRACT

In this paper, a multi-domain technique for 3D elasticity problems is derived from the hybrid boundary node method (Hybrid BNM). The Hybrid BNM is based on the modified variational principle and the Moving Least Squares (MLS) approximation. It does not require a boundary element mesh, neither for the purpose of interpolation of the solution variables nor for the integration of energy. This method can reduce the human-labor costs of meshing, especially for complex construction. This paper presents a further development of the Hybrid BNM for multi-domain analysis in 3D elasticity. Using the equilibrium and continuity conditions on the interfaces, the final algebraic equation is obtained by assembling the algebraic equation for each single sub-domain. The proposed multi-domain technique is capable to deal with interface and multi-medium problems and results in a block sparsity of the coefficient matrix. Numerical examples demonstrate the accuracy of the proposed multi-domain technique.

1. Introduction

Multi-domain (multi-zone) formulations play an important part in numerical analysis when dealing with problems involving interface or dissimilar materials, such as composite materials, etc. Besides, in the case of problems with complicated boundary or a large number of degrees of freedom (DOFs), the use of multi-domain techniques results in a block sparsity matrix and a better computational efficiency may be obtained. The multi-domain techniques for potential and elasticity problems have been widely investigated in boundary element method (BEM) and some other numerical methods. The basic idea of the multi-domain techniques in BEM is that the whole domain is broken up into separate sub-domains, and a boundary integral equation is obtained for each sub-domain. By assembling all contributions of boundary integral equations for each sub-domain, the final system equation can be formed. Ramsak and Skerget [1] present an efficient 3D multi-domain boundary element method for solving problems governed by the Laplace equation. A symmetric Galerkin multi-zone boundary element formulation for elasticity problems is proposed by Layton et al. [2]. Gray and Paulino [3] present a symmetric Galerkin boundary integral method for interface and multi-zone problems. Gao and Davies [4] propose a new technique for 3D elasticity BEM with corners and edges. Gao and Yang [5] derive a boundary integral equation for multi-medium elasticity problems from the boundary-domain integral equation for single medium elasticity problems. A multi-domain boundary contour method (BCM) for interface and dissimilar material problems in elasticity problems is proposed by Phan and Mukherjee [6]. The multi-domain techniques in all of the above-mentioned works are based on the continuity and equilibrium conditions across the interface, namely the continuity of potential (Laplace) or displacement (elasticity) and the equilibrium of flux (Laplace) or traction (elasticity).

In the conventional BEM, the task of mesh generation for complex geometries is often time-consuming and prone to errors even though it only needs discretization of the boundary, and it has difficulties with remeshing in problems involving moving boundaries, large deformations or crack propagation. In the last decades, a new class of numerical methods, namely, the meshless or meshfree methods were developed. One of the main purposes of the meshless techniques is to reduce the human-labor costs required for meshing the domains of complex-shape. There are many meshless techniques have been proposed so far, including the element free Galerkin method (EFG) [7], the meshless local Petrov–Galerkin (MLPG) approach [8], the boundary node method (BNM) [9] and the hybrid boundary node method (Hybrid BNM), etc. The Hybrid BNM is proposed by Zhang et al. [10–14] for potential and elasticity problems and has been developed by Miao et al. [15,16], which combines the MLS approximation [17] scheme with the hybrid displacement variational formula. It not only reduce the spatial dimensions by one like BEM or BNM, but also does not require a boundary element mesh, neither for the purpose of interpolation of the solution variables nor for the integration of energy. In fact, the Hybrid BNM requires only discrete node located on the surface of the domain and its

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doi:10.1016/j.enganabound.2011.01.002

parametric representation. As the parametric representation of created geometry is used on most of CAD packages, it should be possible to exploit their Open Architecture feature, and the required coefficients (representation) can be obtained automatically. Thus, it can reduce the human-labor cost for meshing the domain and can be applied to moving boundaries problems and crack propagation problems efficiently.

A multi-domain formulation of Hybrid BNM for 3D potential problems has been proposed by Zhang et al. [18]. In their work, it was combined with the fast multipole method (FMM) [19], high accuracy and efficiency are observed. However, in Ref. [18], the method proposed by Zhang is for 3D potential problems only. This paper presents a further development of the Hybrid BNM for treating multi-domain problems in 3D elasticity. It is a basic for our future research about the composite material problems, crack problems, etc. And it is a first step for the large scale simulation of composite material with fast algorithm, such as fast multipole method (FMM). In addition, this paper gives the details of equations for arbitrary number of sub-domains.

Based on the continuity of displacement and the equilibrium of traction conditions of nodes on the interfaces, the details of formulas are derived and the final algebraic equation for the multi-domain technique is obtained by assembling the algebraic equation for every single sub-domain. Several numerical examples are present to demonstrate the accuracy of the proposed multi-domain technique.

This paper is organized as follows: In Section 2, the hybrid boundary node method for 3D elasticity problems is reviewed. Section 3 presents an extension of the Hybrid BNM to deal with multi-domain problems in 3D elasticity. Finally, numerical examples are presented in Section 4 to validate the multi-domain technique in this work.

2. The hybrid boundary node method

In this section, the Hybrid BNM for 3D elasticity problems (for detailed discussion see Ref. [10,14]) is reviewed. The Hybrid BNM is based on a modified variational principle [20]. In 3D elasticity problems, the functions in the modified variational principle that assumed to be independent are: displacements $\tilde{u}_i$ and tractions $\tilde{t}_i$ on the boundary and displacements u_i inside the domain. Consider a domain Ω enclosed by $\Gamma=\Gamma_u+\Gamma_t$ with $\bar{u}_i$ and $\bar{t}_i$ are the prescribed displacements and tractions, respectively. The corresponding variational functional Π_{HB} is defined as follows:

$$\Pi_{HB}=\int_\Omega \frac{1}{2}u_{i,j}C_{ijkl}u_{k,l}\,d\Omega-\int_\Gamma \tilde{t}_i(u_i-\tilde{u}_i)\,d\Gamma-\int_{\Gamma_t}\bar{t}_i\tilde{u}_i\,d\Gamma \tag{1}$$

where the boundary displacements $\tilde{u}_i$ satisfies the essential boundary condition, i.e. $\tilde{u}_i=\bar{u}_i$ on Γ_u.

The integral equations can be obtained by $\delta\Pi_{HB}=0$ over the domain and its boundary as follows:

$$\int_\Gamma (t_i-\tilde{t}_i)\delta u_i\,d\Gamma-\int_\Omega \sigma_{ij,j}\delta u_i\,d\Omega=0 \tag{2}$$

$$\int_\Gamma (u_i-\tilde{u}_i)\delta\tilde{t}_i\,d\Gamma=0 \tag{3}$$

$$\int_\Gamma (\tilde{t}_i-\bar{t}_i)\delta\tilde{u}_i\,d\Gamma=0 \tag{4}$$

Eq. (4) will be satisfied if we impose the traction boundary condition, $\tilde{t}_i=\bar{t}_i$ in the same way as the essential boundary condition after the matrices have been computed. So it can be ignored in the following discussion.

The modified variational principle holds both in the whole domain Ω and any sub-domain Ω_I with its boundary Γ_I and L_I. Define the sub-domain Ω_I as an intersection of the domain and a small sphere centered at node $\mathbf{s}_I$, with $\Gamma_I=\partial\Omega_I\cap\Gamma$ and $L_I=\partial\Omega_I-\Gamma_I$, respectively. We can obtain the following weak forms for the sub-domains and its boundaries to replace Eqs. (2) and (3):

$$\int_{\Gamma_I+L_I}(t_i-\tilde{t}_i)h_I\,d\Gamma-\int_{\Omega_I}\sigma_{ij,j}h_I\,d\Omega=0 \tag{5}$$

$$\int_{\Gamma_I+L_I}(u_i-\tilde{u}_i)h_I\,d\Gamma=0 \tag{6}$$

where h_I is a weight function.

The displacements $\tilde{u}$ and tractions $\tilde{t}$ at the boundary Γ are approximated by the MLS approximation as follows:

$$\tilde{u}(\mathbf{s})=\sum_{J=1}^{n}\Phi_J(\mathbf{s})\hat{u}_J \tag{7}$$

$$\tilde{t}(\mathbf{s})=\sum_{J=1}^{n}\Phi_J(\mathbf{s})\hat{t}_J \tag{8}$$

where n is the number of nodes for MLS approximation which located on the surface; $\hat{u}_J$ and $\hat{t}_J$ are nodal values, and $\Phi_J(\mathbf{s})$ is the shape function of the MLS approximation, corresponding to node $\mathbf{s}_J$ (full details refer to [10,14]).

In Eqs. (5) and (6), $\tilde{u}_i$ and $\tilde{t}_i$ on Γ_I can be expressed by Eqs. (7) and (8) since Γ_I is a portion of Γ, but $\tilde{u}_i$ and $\tilde{t}_i$ on L_I have not been defined yet. In order to solve this problem, we deliberately select h_I such that all integrals over L_I vanished. This can be easily accomplished by using the weight function in the MLS approximation for h_I, with replacing the radius of the support of the weight function by the radius r_I of the sub-domain Ω_I (full details refer to [10,14]).

With h_I vanishing at L_I, the integrals over L_I in Eqs. (5) and (6) are zero and the two Eqs. can be rewritten as

$$\int_{\Gamma_I}(t_i-\tilde{t}_i)h_I\,d\Gamma-\int_{\Omega_I}\sigma_{ij,j}h_I\,d\Omega=0 \tag{9}$$

$$\int_{\Gamma_I}(u_i-\tilde{u}_i)h_I\,d\Gamma=0 \tag{10}$$

The $\mathbf{u}$ and $\mathbf{t}$ inside the domain can be approximated by fundamental solutions as

$$\mathbf{u}=\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}=\sum_{J=1}^{N}\begin{bmatrix}u^J_{11}&u^J_{12}&u^J_{13}\\u^J_{21}&u^J_{22}&u^J_{23}\\u^J_{31}&u^J_{32}&u^J_{33}\end{bmatrix}\begin{Bmatrix}x^J_1\\x^J_2\\x^J_3\end{Bmatrix} \tag{11}$$

$$\mathbf{t}=\begin{Bmatrix}t_1\\t_2\\t_3\end{Bmatrix}=\sum_{J=1}^{N}\begin{bmatrix}t^J_{11}&t^J_{12}&t^J_{13}\\t^J_{21}&t^J_{22}&t^J_{23}\\t^J_{31}&t^J_{32}&t^J_{33}\end{bmatrix}\begin{Bmatrix}x^J_1\\x^J_2\\x^J_3\end{Bmatrix} \tag{12}$$

where u^J_{ij} and t^J_{ij} are the fundamental solutions; x^J_i are unknown parameters; N is the total number of boundary nodes for approximation of $\mathbf{u}$ and $\mathbf{t}$ inside the domain.

From the fundamental solution, one can see that it leads to singularities in the integrals of Eqs. (9) and (10) and there exists domain integrals in Eq. (9) if the field point Q and source point $\mathbf{s}_J$ are coincide. The singularities in Eq. (10) are weak and can be evaluated directly. However, the integrals in Eq. (9) are strong singularities. In order to avoid direct numerical integration of the strong singularities and domain integrals, the rigid body movement is utilized (full details see Ref. [14]). So, in the following equations, the domain integrals are ignored.