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Efficient numerical models for the prediction of acoustic wave propagation in the vicinity of a wedge coastal region

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ABSTRACT

In this paper, numerical frequency domain formulations are developed to simulate the 2D acoustic wave propagation in the vicinity of an underwater configuration which combines two sub-regions: the first one consists of a wedge with rigid seabed and free surface, and the second one is assumed to have a rigid flat bottom and a free flat surface.

The problem is solved using two different numerical methods: the Boundary Element Method (BEM) and the Method of Fundamental Solutions (MFS). Two models are developed by using a sub-region technique, where only the vertical interface between sub-regions of different geometries has to be discretized. These formulations incorporate Green's functions that take into account the presence of flat rigid and free surfaces and of a wedge. Green's functions are defined using two approaches: the image source method is used to model the rigid flat bottom and free flat interface, whereas the response provided by the wedge sub-region is based on a normal mode solution. Additionally, a MFS and a BEM model are also implemented which require the discretization of the sloping rigid seabed of the wedge, therefore making use of Green's functions for a rigid flat bottom and a free surface (using the image source method).

A detailed discussion on the performance of these formulations is performed, with the aim of finding an efficient formulation to solve the problem. It is found that the model based on the MFS and on the sub-region technique has a significantly lower computational cost and is stable, therefore being the most suitable for the analysis of acoustic wave propagation in the studied configurations.

1. Introduction

Computational methods for determining the sound field in an acoustic medium have evolved over the past decades. In the specific field of underwater acoustics, many methods have been applied with great success, as documented in the excellent reference work [1], all of them with specific limitations. Some of the most well-known and broadly applied methods are based on acoustic ray theory, normal mode analysis (pioneered by the works of Pekeris [2]), simplified parabolic equations (initially introduced in this field by Hardin and Tappert [3]) or Green's functions for layered media (as defined, for example, in the works of Schmidt and Tango [4] or Tadeu et al. [5,6]).

The more exact wave theory, together with modern high-speed computing infrastructures and the advances in numerical physics, allowed the development of different and more accurate approaches for some specific problems in underwater acoustics, including models based on the well-established finite difference, finite element and boundary element numerical methods.

An important early work on acoustic scattering in the open ocean using the Boundary Element Method, by Dawson and Fawcett [7], takes the waveguide surfaces to be flat, except for a compact area of deformation where the acoustic scattering takes place. An application of the Boundary Element Method (BEM) using a hybrid model which combines the standard method in an inner region with varying bathymetry and an eigenfunction expansion in the outer region of constant depth was subsequently presented by Grilli et al. [8].

Santiago and Wrobel [9,10] implemented the sub-region technique in boundary element formulation for the analysis of two-dimensional acoustic wave propagation in a shallow water region with irregular seabed topography. In their approach the bottom and surface boundaries of the regions are modeled using Neumann and Dirichlet conditions, allowing for the use of Green's functions that satisfy either the free surface boundary condition or both the boundary conditions on the free surface and rigid bottom.

In a number of acoustic environments, the geometry of the propagation domain can be assumed to be constant in one

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direction. When the acoustic excitation is modeled as a 3D source, these problems are usually called 2.5D, thus the 3D acoustic wave equation can be mathematically manipulated to obtain the frequency-domain solution as a summation of simpler 2D problems [11]. Boundary element models have also been developed using this technique to compute the pressure field in ocean environments [12,13].

One of the difficulties of boundary element methods in the analysis of acoustic environments occurs when more complex geometries are analyzed, for which case it may be necessary to perform large discretization schemes, leading to high computational effort. One way of avoiding these large discretizations is incorporating appropriate Green's functions to account for part of the boundaries, such as a free surface. However, in many cases, these functions are built as convergent series, and to take advantage of their use in numerical codes, it may be necessary to previously evaluate their convergence requirements.

More recently, researchers have also focused their attention to a different class of numerical methods, usually designated as meshless methods, which do not require explicit domain or boundary discretization. One such technique is the Method of Fundamental Solutions (MFS) [14–16]. Mathematically, the MFS is rather simple, but it also requires previous knowledge of Green's functions of the propagation domains. In fact, very little can be found in the literature concerning the application of such methods to underwater acoustics, although its development for applications in the analysis of sound propagation in underwater environments may be an interesting approach. For such methods, the use of Green's functions that automatically satisfy the specific boundary conditions may further decrease computational cost.

In the present paper the authors aim to analyze the efficiency and accuracy of different numerical formulations developed to allow for the computation of the acoustic wave propagation in a specific underwater acoustics configuration. These formulations may incorporate Green's functions that allow reducing the discretization needs. Two different numerical methods are used here: the first one is the Boundary Element Method (BEM) and the second is the Method of Fundamental Solutions (MFS). In all the cases, the analyzed region is assumed to be two-dimensional, simulating underwater acoustic problems which have little variation in the long shore direction, and is divided in two sub-regions: the first one is a flat waveguide, composed of a flat rigid bottom and a flat free surface; the second is defined by a wedge, with rigid bottom and free surface.

Four formulations are discussed. Two models are based on BEM and MFS, respectively, requiring the definition of two sub-domains where appropriate Green's functions can be determined, and thus only requiring the interface between sub-regions to be discretized. In the first sub-domain, which assumes a flat rigid bottom and a flat free surface, Green's functions obtained using the Image Source Method are used. In the second one, defining a wedge, Green's functions are obtained using an approach based on the sum of normal modes [17]. The other two models also make use of the MFS and BEM, but require discretization of the inclined rigid bottom that forms the wedge. These models assume only one region and make use of adequate Green's functions based on the Image Source Method.

The organization of this paper is as follows: first, the governing equation of the problem is presented; there follows the description of the developed Boundary Element Method formulation and of the Method of Fundamental Solutions; Green's functions used are then defined, followed by a convergence analysis performed for the case of the wedge; after this, the proposed models are verified by comparing the results they provide against each other; the performance and accuracy of the models are discussed in detail, in order to evaluate their advantages and limitations; finally, an example application is presented, illustrating the applicability of the proposed formulation.

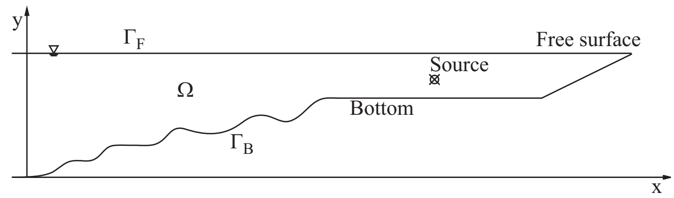

Fig. 1. Geometry of the problem.

2. Governing equation of the problem

Consider the problem of acoustic wave propagation in a region Ω of infinite extent along the z direction, with irregular rigid seabed topography and flat free surface, as shown in Fig. 1.

If the source of acoustic disturbance is time-harmonic, the sound velocity is constant and the medium in the absence of perturbations is quiescent, the governing Helmholtz equation for this problem can be written as

$$\nabla^2\phi + k^2\phi = -\sum_{l=1}^{nf} Q_l \delta(\xi_l^f, \xi), \quad \text{in region } \Omega \tag{1}$$

where nf is the number of sources in domain; ϕ is the velocity potential; Q_l is the magnitude of the existing sources ξ_l^f located at $(x_{\xi_l^f}, y_{\xi_l^f})$; ξ is a domain point located at (x_ξ, y_ξ); $\delta(\xi_l^f, \xi)$ is the Dirac delta generalized function; and $k=\omega/c$ is the wave number, with ω being the angular frequency and c the speed of sound in the medium.

The described problem is subjected to the following boundary conditions:

- *Dirichlet condition*

$$\phi(\mathbf{x}) = 0 \quad \text{in } \Gamma_F \tag{2}$$

- *Neumann condition*

$$\frac{\partial\phi}{\partial \mathrm{n}}(\mathbf{x}) = 0 \quad \text{in } \Gamma_B \tag{3}$$

- *Sommerfeld radiation condition at infinity*

$$\lim_{\mathrm{x}\to\infty} \frac{\partial\phi}{\partial n}(\mathbf{x}) - ik\phi(\mathbf{x}) = 0 \tag{4}$$

where $\mathbf{x}$ is the field point located at (x, y), Γ_F is the boundary of the free surface, Γ_B is the boundary of the bottom, n is the unit outward normal vector and $i=\sqrt{-1}$.

3. Numerical formulations

The previously described wave propagation problem is solved using four numerical formulations, based on the Boundary Element Method and on the Method of Fundamental Solutions. In the following sub-sections these formulations are described.

3.1. Boundary Element Method

According to Green's second identity, Eq. (1) can be transformed into the following boundary integral equation:

$$C(\xi)\phi(\xi) = \int_\Gamma G(\xi,\mathbf{x}) \frac{\partial\phi}{\partial \mathrm{n}}(\mathbf{x}) d\Gamma - \int_\Gamma \frac{\partial G(\xi,\mathbf{x})}{\partial \mathrm{n}} \phi(\mathbf{x}) d\Gamma + \sum_{l=1}^{nf} Q_l G(\xi_l^f, \xi) \tag{5}$$