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Numerical simulation of forward problem for electrical capacitance tomography using element-free Galerkin method

Lifeng Zhang *, Pei Tian, Xiuzhang Jin, Weiguo Tong

School of Control Science and Engineering, North China Electric Power University, Baoding 071003, China

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ABSTRACT

Electrical capacitance tomography (ECT) is a new measurement technology, which is often used to identify two-phase/multi-phase flow regime and investigate the solid distribution in circulating fluidized bed. It is composed of forward problem and inverse problem. Usually, forward problem is solved using finite element method (FEM). Element-free Galerkin method (EFGM) is one of the meshless methods developed recently. In order to obtain the numerical solution using EFGM, a shape function is constructed by moving least-squares (MLS) approximation, a variational equation weak form of the studied problem is used to deduce the discrete equation, and Lagrange multipliers are used to satisfy essential boundary conditions. In EFGM, only nodal data are necessary compared with FEM. In this paper, EFGM was used to solve forward problem and simulation results showed EFGM has high accuracy and its post-processing is easy.

1. Introduction

There exist many two-phase flow systems in power plant, petroleum plant and chemical process [1]. Two-phase flow regime identification and solid concentration measurement are always the important research problems. ECT is a kind of process tomography technology and provides a new way to solve the problems [2–4].

ECT technique includes forward problem and inverse problem [5]. Usually, forward problem is solved using FEM, which causes some problems as follows:

1. A mesh is needed before computing. It is time-consuming to prepare data, especially for three-dimensional calculation. When the FEM is used in adaptive calculation, the mesh must be updated often, which cause a computing bottleneck.
2. The field function is not continuous.
3. There are special demands for mesh element. For example, the triangle element cannot have great large obtuse angle or small sharp angle.

In recent years, more and more attention has been paid to researches on the meshless method, which makes it a hot direction of computational mechanics [6–8]. The main objective for the development of meshless methods is to eliminate even a small degree complexity of the analysis method by making approximation, which is entirely based on nodes and not on elements. The large deformation and crack growth problems can be simulated with the method without the re-meshing technique. The EFGM is regarded as a meshless method, which employs the MLS approximation to approximate the unknown function [9–12]. The potential advantages of this method compared to the FEM are [13]:

1. Only a set of nodes and a boundary description are needed to develop the discrete equations while the element mesh is unnecessary.
2. The dependant variable and its gradient are continuous in the entire domain and post-processing to obtain smooth gradient field is totally unnecessary.
3. Adaptivity in problems with deformations becomes relatively easy, since it is necessary to add nodes.

In this paper, the author presents the implementation of the EFGM for forward problem computations of ECT. Numerical simulation results were presented in comparison with the FEM solutions.

2. Mathematical formulations

2.1. MLS approximation

The discretization of the governing equations by EFGM requires MLS interpolation functions that are made up of three components: a weight function associated with each node, a monomial basis function and a set of non-constant coefficients. The weight function is non-zero over a small neighborhood at a particular node, called support of the node [14,15].

* Corresponding author. Tel.: +86 312 7522 244; fax: +86 312 7522 507.
E-mail address: hdlfzhang@126.com (L. Zhang).

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Using MLS approximation, the unknown function $u(x, y)$ is approximated by $u^h(x, y)$ over the two-dimensional domain Ω [16]

$$u^h(x,y) = \sum_{i=1}^{m} p_i(x,y)a_i(x,y) = \mathbf{P}^T(x,y)\mathbf{a}(x,y) = \mathbf{P}^T(X)\mathbf{a}(X) \tag{1}$$

where m is the number of terms in the basis, $p_i(x, y)$ is monomial basis function, $a_i(x, y)$ are non-constant coefficients, $X=[x, y]$ and $\mathbf{P}^T(X)=[1, x, y]$.

The coefficients $a_i(x, y)$ are found by minimizing the quadratic functional $J(X)$ given by

$$J(X) = \sum_{i=1}^{n} w(X-X_i)[u_L^h(X,X_i)-u_i]^2 = \sum_{i=1}^{n} w(X-X_i)[\mathbf{P}^T(X_i)\mathbf{a}(X)-u_i]^2 \tag{2}$$

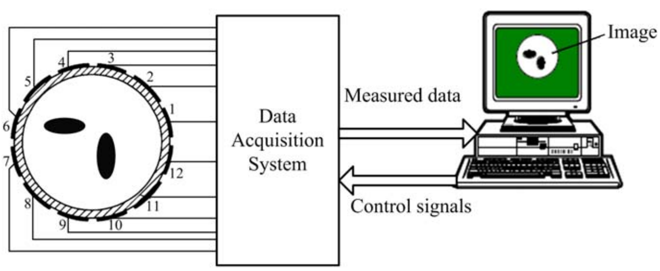

Fig. 1. Typical ECT system with a 12-electrode sensor.

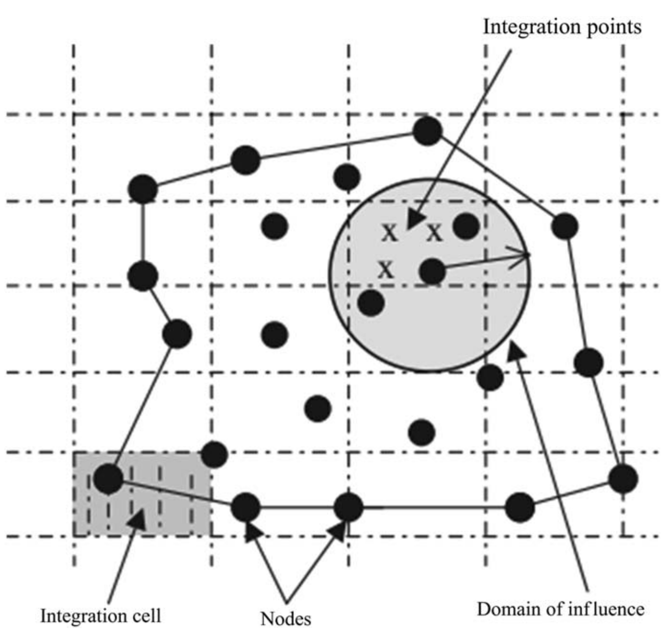

Fig. 2. Geometrical model of EFGM.

Table 1
Simulation parameters.

Parameters	Value
Inner diameter	60mm
Outer diameter	62.5 mm
Screen diameter	75mm
The relative permittivity	1
Excitation voltage	10 V

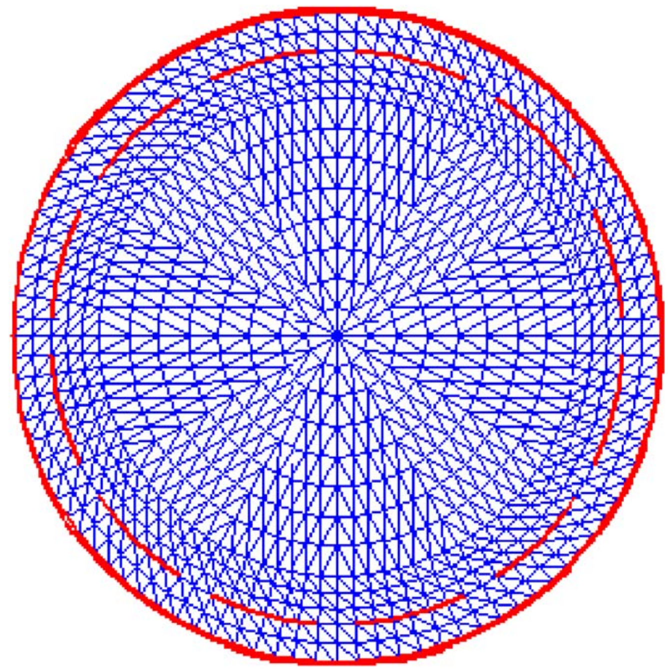

Fig. 3. Diagram of mesh grid.

Table 2
Calculated potentials using EFGM and FEM at location $x=0$, $0 \leqslant y \leqslant 75$ mm.

y (mm)	EFGM solution (V)	FEM solution (V)
0	0.8108	0.8082
12	0.8436	0.8335
24	0.7630	0.7517
36	0.5814	0.5704
48	0.3310	0.3298
60	0.0910	0.0828
65	0.0298	0.0242
70	0.0097	0.0090
75	0.0	0.0

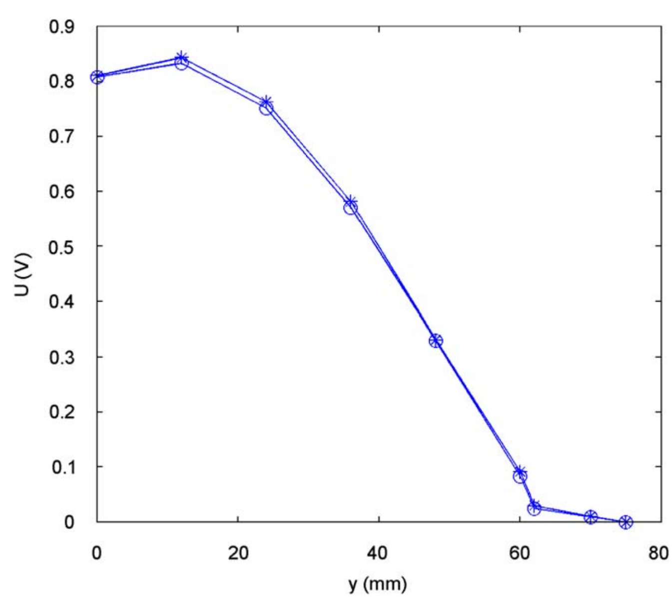

Fig. 4. Calculated potentials using EFGM and FEM at location $x=0$, $0 \leq y \leq 75$ mm.