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Three-dimensional static analysis of thick functionally graded plates by using meshless local Petrov–Galerkin (MLPG) method

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ABSTRACT

In this paper, a version of meshless local Petrov–Galerkin (MLPG) method is developed to obtain three-dimensional (3D) static solutions for thick functionally graded (FG) plates. The Young's modulus is considered to be graded through the thickness of plates by an exponential function while the Poisson's ratio is assumed to be constant. The local symmetric weak formulation is derived using the 3D equilibrium equations of elasticity. Moreover, the field variables are approximated using the 3D moving least squares (MLS) approximation. Brick-shaped domains are considered as the local sub-domains and support domains. In this way, the integrations in the weak form and approximation of the solution variables are done more easily and accurately. The proposed approach to construct the shape and the test functions make it possible to introduce more nodes in the direction of material variation. Consequently, more precise solutions can be obtained easily and efficiently. Several numerical examples containing the stress and deformation analysis of thick FG plates with various boundary conditions under different loading conditions are presented. The obtained results have been compared with the available analytical and numerical solutions in the literature and an excellent consensus is seen.

1. Introduction

Functionally graded materials (FGMs) are advanced composites in which the material properties vary smoothly and continuously by a predetermined function. Generally, those are composed of ceramics and metals so that the material properties such as the Young's modulus vary from the metallic side to the ceramic one. Initially, they were designed as thermal barrier coatings in space applications. However, recently FGMs have gained lots of applications in nuclear reactors [1], dental and medical implants [2], piezoelectric and thermo electric devices [3–5] and fire retardant doors [6]. Investigation of the mechanical behavior of new materials such as FGMs is an attractive research area in mechanics. Many researchers have studied the mechanical behavior of FGMs having various geometries and loading conditions. Among the geometries, which are considered to analyze, the plates are the most significant due to numerous applications in engineering structures.

It should be mentioned that the plate is a three-dimensional (3D) structure in which one dimension is relatively much smaller than the other dimensions. In 2D theories such as the classical plate theory (CPT), in the first order shear deformation plate theory (FSDT) and the higher order shear deformation plate theory (HSDT), various assumptions are made to obtain a 2D formulation for plates. Obviously, it is easier to find a solution for the plate problems in 2D formulation. However, some errors occur in solutions due to the assumptions considered in 2D theories. With the increase in the thickness of plates, the errors amplify. If no simplifying assumptions are considered, the 3D elasticity equations must be used for elastic analysis of plates. The 3D solution does not involve any limitation of 2D solutions. It is obvious that if 3D solutions are attainable, it will be more accurate than the solutions obtained by the mentioned 2D theories.

Analytical and numerical methods have been used for 3D analyses of plates [7–13]. Analytical methods are applicable for some simple cases and generally, the analytical solutions cannot be found for plates with complicated geometries and boundary conditions. Consequently, numerical methods such as the finite element method (FEM) and differential quadrature (DQ) method are the alternatives that have been employed for 3D analysis of plates. Recently, meshless methods, which do not need burdensome effort of mesh generation have gained lots of attention. A variety of these methods have been developed, such as the element-free Galerkin (EFG) method [14], the reproducing kernel particle method (RKPM) [15], hp-clouds [16], and the partition of unity method (PUM) [17], which have been successfully applied for the solutions of some engineering problems. However, most of

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these methods are not truly meshless, because, they use background cells for the evaluation of integrals in the weak formulation of problems.

The meshless local Petrov–Galerkin (MLPG) method proposed by Atluri et al. [18–21] considers local weak formulation instead of the global weak formulation, and does not require a background mesh for evaluation of the integrals. In the MLPG method, the integrals of the weak form are evaluated over local sub-domains that partly cover each other. The trial and test functions are chosen from totally different functional spaces. Furthermore, the physical size of the test and trial domains is not necessary to be the same, which makes the MLPG a very flexible method. Based on the concept of the MLPG, six different methods have been introduced, which are labeled as MLPG1–MLPG6 [21]. These six methods differ due to the type of test function considered in the weak formulation. Among them, the MLPG1 and MLPG5 seem to be the most promising formulations; so they are used in the present study. In the MLPG1 a namely fourth-order spline function is considered as test function, while the Heaviside step function is employed as test function in MLPG5. It is noticeable that the MLPG5 does not involve any domain integration or singular integrals. The MLPG methods have been employed in a wide range of applications, for example elasto-statics [22], elasto-dynamics [23], fluid mechanics [24], convection–diffusion problems [25], thermoelasticity [26], beam problems [27,28], plate problems [29–31], FGM problems [32,33], fracture mechanics [34–36], and strain gradient theory [37]. So far, the applications of the MLPG method are mostly limited to 2D problems. The major reason for the 3D analysis is considered less, related to difficulties in evaluating the integrals over the local sub-domains, especially when a sub-domain intersects the global boundary of problem.

However, significant efforts have been carried out to develop the MLPG method to solve 3D problems. Li et al. [38] applied a combination of MLPG2 and MLPG5 for the solution of two classical 3D problems, viz., the Boussinesq problem and the Eshelby's inclusion problem. They used MLPG5 for nodes inside the domain and MLPG2 for nodes on the boundary of the problem domain. Han and Atluri [39] used the MLPG method to solve 3D elastic fracture problems. Also, Han and Atluri [40] developed three kinds of the MLPG methods using different test functions for analysis of thick beams and spheres and examined the performance of the methods. They used Delaunay triangulation algorithm and introduced a suitable mapping procedure for evaluating the integrals over the spherical sub-domains. The integrals on the volume of the sub-domains were calculated by dividing the domains into cone shapes. The MLPG domain discretization method has been employed to 3D elasto-dynamic problems of impact and fragmentation by Han and Atluri [41]. Vaghefi et al. [42] developed two different 3D MLPG procedures including MLPG1 and MLPG5 for the elasto-static analysis of thick rectangular plates with various boundary conditions. Brick-shaped domains are considered as sub-domains and support domains.

The purpose of the present paper is to develop a complete 3D MLPG procedure for the elasto-static analysis of thick FG plates with various boundary conditions. The Young's modulus of the plate is assumed to vary exponentially through the thickness, and the Poisson's ratio is assumed to be constant. The local symmetric weak form (LSWF) is used to formulate the problem and 3D MLS approximation is used to approximate the field variables. To impose the essential boundary conditions, the penalty method is adopted. Since the global domain of the problem is cubic, brick-shaped domains are considered as sub-domains and support domains for evaluating the integrals of the weak form and approximating the solution variables, respectively. Moreover, the approach used for the construction of the shape and the test functions makes it possible to add nodes in any direction of the plate (through length, width and thickness) depending upon the required accuracy. Therefore, sufficient number of nodes has been added in the thickness direction of the plate to represent the normal and shear stress variations through the thickness with good accuracy. Consequently, more precise solutions can be obtained easily and efficiently. In order to illustrate the accuracy of the proposed approach, several numerical examples are presented and the results are compared with the known solutions in the literature.

2. MLPG formulation for 3D elasticity

The 3D elasto-static equilibrium equations in a domain of the volume Ω, which is bounded by the surface Γ, are given by

$$\sigma_{ij,j}+b_i=0, \text{ in } \Omega \tag{1}$$

where σ_{ij} is the stress tensor and b_i is the body force vector. The indices i, j which take the values of 1, 2, 3 refer to the Cartesian coordinates x, y, z, respectively. The boundary conditions are assumed to be

$$u_i=\overline{u}_i \text{ on } \Gamma_u, \tag{2a}$$

$$\sigma_{ij}n_j=\overline{t}_i \text{ on } \Gamma_t \tag{2b}$$

where u_i and t_i represent the displacement components and the surface traction components, respectively. Γ_u is the boundary with the prescribed displacement $\overline{u}_i$ while Γ_t is the boundary with the prescribed traction $\overline{t}_i$. n_j denotes the unit outward vector normal to the boundary Γ.

The weak formulation is constructed over the local sub-domains, which are located inside the global domain Ω. The local sub-domains may overlap with each other and must cover the whole global domain. Various shapes with different sizes can be chosen as sub-domains, but appropriate ones should be considered to obtain precise outcomes. Since the global domain of the rectangular plate is hexahedral, brick-shaped local domains are considered as sub-domain and support domain (see Fig. 1). Selecting the brick-shaped local domain makes the mapping procedure easy and no special treatment is needed when the local sub-domain intersects the global boundary.

2.1. Local symmetric weak form for 3D elastic body

The generalized local weak form of the equilibrium equations over a local sub-domain around node I is written as follows:

$$\int_{\Omega_s^I}(\sigma_{ij,j}+b_i)v_I\,d\Omega_s-\alpha\int_{\Gamma_{su}^I}(u_i-\overline{u}_i)v_I\,d\Gamma=0 \tag{3}$$

where u_i, v_I are the trial and the test functions, respectively and can be chosen from different functional spaces. In Eq. (3), the term

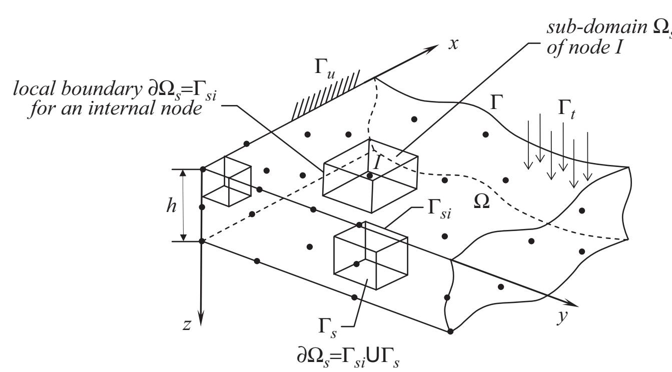

Fig. 1. Brick-shaped sub-domains in the global domain of a rectangular plate.