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Three-dimensional stress intensity factors of a central square crack in a transversely isotropic cuboid with arbitrary material orientations

Chao-Shi Chen [a,*], Chia-Hau Chen [a], Ernian Pan [b]

[a] Department of Resources Engineering, National Cheng Kung University, Tainan 701, Taiwan
[b] Department of Civil Engineering, University of Akron, Akron, OH 44325-3905, USA

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ABSTRACT

In this paper, we present the dual boundary element method (dual-BEM) or single-domain BEM to analyze the mixed three-dimensional (3D) stress intensity factors (SIFs) in a finite and transversely isotropic solid containing an internal square crack. The planes of both the transverse isotropy and square crack can be oriented arbitrarily with respect to a fixed global coordinate system. A set of four special nine-node quadrilateral elements are utilized to approximate the crack front as well as the outer boundary, and the mixed 3D SIFs are evaluated using the asymptotic relation between the SIFs and the relative crack opening displacements (COD) via the Barnett–Lothe tensor.

Numerical examples are presented for a cracked cuboid which is transversely isotropic with any given orientation and is under a uniform vertical traction on its top and bottom surfaces. The square crack is located in the center of the cuboid but is oriented arbitrarily. Our results show that among the selected material and crack orientations, the mode-I SIF reaches the largest possible value when the material inclined angle $\psi_1 = 45^\circ$ and dig angle $\beta_1 = 45^\circ$, and the crack inclined angle $\psi_2 = 0^\circ$ and dig angle $\beta_2 = 0^\circ$. It is further observed that when the crack is oriented vertically or nearly vertically, the mode-I SIF becomes negative, indicating that the crack closes due to an overall compressive loading normal to the crack surface. Variation of the SIFs for modes II and III along the crack fronts also shows some interesting features for different combinations of the material and crack orientations.

1. Introduction

Rock fracture mechanics is essentially an extension of the classic fracture mechanics in solids [1,2], which recognizes the importance of the stress intensity near a crack tip. Irwin [2] introduced the stress intensity factors (SIFs) to describe the stress and displacement fields near a crack tip. As is well known, there are three basic crack propagation modes in the fracture process: opening (mode I), sliding (mode II), and tearing (mode III), and furthermore, mechanical safety of a solid elastic structure can be analyzed based on these SIFs. Therefore, determination of SIFs near the crack front in linear elastic fracture mechanics has been always an interesting but challenging task. While most previous studies in SIFs were focused on one or two fracture modes, mixed three-dimensional (3D) modes need to be considered as materials could mostly fail under combined tensile/compressive, shearing, and tearing loads. For 3D isotropic elastic materials, Singh et al. [3] obtained the SIFs using the concept of a universal crack closure integral. For transversely isotropic, orthotropic, and anisotropic solids, Pan and Yuan [4] presented the general relationship between the SIF and the relative crack opening displacement (COD). Lazarus et al. [5] compared the calculated SIFs with experimental results for brittle solids under mixed mode I–III or I–II–III loadings. The 3D SIFs were also calculated by Zhou et al. [6] using the variable-order singular boundary element. More recently, Yue et al. [7] employed the dual boundary element method (dual-BEM) in their calculation of the 3D SIFs of an inclined square crack within a bi-material cuboid. We point out that the dual-BEM was originally proposed by Hong and Chen [8] as reviewed in [9]. Other recent representative works in this direction are those by Ariza and Dominguez [10], Liu et al. [11], Hatzigeorgiou and Beskos [12], Partheymüller et al. [13], Popov et al. [14], Zhao et al. [15], Lo et al. [16], dell'Erba and Aliabadi [17], and Blackburn [18]. The weakly singular and weak-form integral equation method recently proposed by Rundamornrat [19] and Rungamornrat and Mear [20] is also very efficient in crack analysis in anisotropic media. Besides the analytical (integral equation) and BEM methods [21], other common methods, such as the finite difference (FD) [22–24] and the finite element (FE) [25,26], were also applied for 3D SIF analysis. Since both the FD and FE methods require discretization of the whole problem domain,

* Corresponding author.
E-mail addresses: chencs@mail.ncku.edu.tw (C.-S. Chen), pan2@uakron.edu (E. Pan).

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these methods could be time consuming and more expensive than the BEM in fracture analysis.

In this paper, we apply the dual-BEM or the single-domain BEM to analyze the mixed 3D SIFs in a finite and transversely isotropic solid containing an internal square crack. Both the transversely isotropic plane and square crack plane can be oriented arbitrarily with respect to a fixed global coordinate system. A set of special nine-node quadrilateral elements are utilized to approximate the crack surface as well as the outer boundary, and the mixed 3D SIFs are evaluated using the asymptotic relation between the SIFs and the relative CODs via the Barnett–Lothe tensor. Our numerical examples show clearly the strong dependence of 3D SIFs on both the material and crack orientations, and these results could be useful in fracture analysis and design of anisotropic elastic solids.

The paper is organized as follows: in Section 2, we briefly present the required basic equations, including the related local and global coordinate systems. In Section 3, the two important BEM equations are presented for the modeling of a cracked 3D anisotropic cuboid: One is the displacement BEM and another is the traction BEM. The special crack front elements and the corresponding formulation for the mixed SIF calculation are presented in Section 4, and detailed numerical results are discussed in Section 5. Finally, conclusions are drawn in Section 6.

2. Basic equations

We consider a transversely isotropic elastic finite domain with arbitrarily oriented transverse isotropy plane. Inside this domain, there is a central square crack, which is also oriented arbitrarily. First, shown in Fig. 1 is the relation between the global coordinates (x,y,z) or (x_1,x_2,x_3) and the local transversely isotropic material coordinate system x', y', and z', where z' is along the symmetry axis of the material, and $(x'-y')$ is parallel to the isotropic plane. The inclined angle ψ_1 is defined as the angle between the global horizontal plane and the isotropic plane of the material, and the dip orientation β_1 is defined as the angle between the inclined angle plane and the global y-axis.

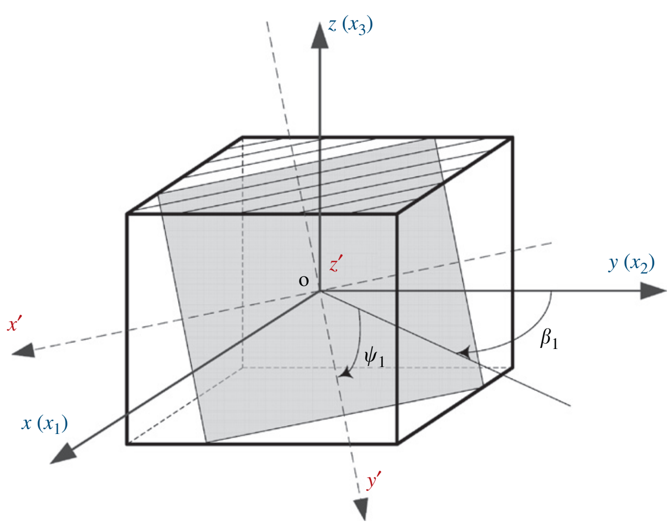

Fig. 1. Relation between the local (x',y',z') and global (x,y,z) coordinate systems where the local z'-axis is along the symmetry axis of the transversely isotropic material. In other words, the local (x',y')-plane is parallel to the isotropic plane of the material; ψ_1 is the inclined angle between the global horizontal plane (x,y) and the local isotropic plane (x',y'); β_1 is the dip orientation between the global y-axis and the include plane.

It is obvious that the transformation between the local (x',y',z') and global (x,y,z) coordinates can be described by the following relation:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\beta_1 & -\sin\beta_1 & 0 \\ \cos\psi_1\sin\beta_1 & \cos\psi_1\cos\beta_1 & -\sin\psi_1 \\ \sin\psi_1\sin\beta_1 & \sin\psi_1\cos\beta_1 & \cos\psi_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{1}$$

To solve the problem using the BEM formulation, we first present the governing equations in linear elasticity.

(1) Equations of equilibrium:

$$\sigma_{ij,j} + b_i = 0, \quad i,j = 1,2,3 \tag{2}$$

where σ_{ij} is the stress tensor; b_i the body force component; and subscript "$,j$" denotes partial differentiation with respect to the global coordinates x, y, and z.

(2) Constitutive relation:

$$[\varepsilon] = [a][\sigma] \tag{3}$$

where

$$[\varepsilon] = [\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{23}, 2\varepsilon_{13}, 2\varepsilon_{12}]^t \tag{4}$$

is the strain in the column matrix form, and

$$[\sigma] = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}]^t \tag{5}$$

is the stress in the column matrix form. Also in Eq. (3), $[a]$ is the elastic compliance matrix of the anisotropic elastic solid. For the transversely isotropic material, there are five independent elastic parameters (E, E', ν, ν', and G') in the matrix $[a]$. The definitions of these moduli are: E and E' are the Young's moduli in the plane of transverse isotropy and in a direction normal to it, respectively; ν and ν' are the Poisson's ratios characterizing the lateral strain response in the plane of transverse isotropy to a stress acting parallel and normal to it, respectively; G' is the shear modulus normal to the plane of transverse isotropy. Furthermore, the shear modulus G in the plane of transverse isotropy is equal to $E/(2(1+\nu))$. For the transversely isotropic material oriented arbitrarily with respect to the global coordinate system, the matrix $[a]$ will also be a function of the inclined angle ψ_1 and dip orientation β_1. As an example, Appendix A lists the elements of $[a]$ when $\beta_1 = 0$ [27]. It is further observed from Appendix A that the coefficients a_{15}, a_{25}, a_{35}, and a_{46} are zero when ψ_1 is equal to 0° or 90°.

(3) Strain–displacement relation:

$$\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i}), \quad i,j = 1,2,3 \tag{6}$$

where u_i are the elastic displacements.

3. Single-domain boundary integral equations

We now present the basic relations in the boundary element analysis for fracture problems in linear elasticity. It is based on the dual-BEM [8,9] or the single-domain BEM [28] approach.

We assume that the finite domain under consideration is free of any body force (i.e., $b_i = 0$ in Eq. (2)) and is bounded by an outer boundary S with given boundary conditions. Inside its domain, there is a crack described by its surface Γ (where $\Gamma = \Gamma^+ = -\Gamma^-$, with superscripts "+" and "−" denoting the positive and negative sides of the crack). We further assume that the tractions on both sides of the crack are equal and opposite. Then the single-domain BEM formulation consists of the following displacement and traction boundary integral equations [29].