Engineering Analysis with Boundary Elements 33 (2009) 147–158

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Evaluation of the degenerate scale for BIE in plane elasticity and antiplane elasticity by using conformal mapping

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ARTICLE INFO

Article history:
Received 16 November 2007
Accepted 30 May 2008
Available online 29 July 2008

Keywords:
Boundary integral equation
Degenerate scale
Conformal mapping technique
Closed form solution
Plane elasticity: Antiplane elasticity

ABSTRACT

For a better understanding for the formulation of the degenerate scale problem by using the complex variable, preliminary knowledge is introduced. Formulation for the degenerate scale problem is based on the direct usage of the complex variable and the conformal mapping. After using the conformal mapping, the vanishing displacement condition is assumed on the boundary of unit circle. The complex potentials on the mapping plane are sought in a form of superposition of the principal part and the complementary part. The principal part of the complex potentials is given beforehand, and the complementary part plays a role for compensating the displacement along the boundary from the principal part. After using the appropriate complex potentials, the boundary displacement becomes one term with the form of $g(R)-c$ ($g(R)$ a function of R), where R denotes a length parameter. By letting the vanishing displacement on the boundary, or $g(R)-c=0$, the degenerate scale "R" is obtained. For four cases, the elliptic contour, the triangle contour, the square contour and the ellipse-like contour, the degenerate scales are evaluated in a closed form. For the case of antiplane elasticity, similar degenerate scale problems are solved.

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1. Introduction

The boundary integral equation (abbreviated as BIE) was widely used in elasticity, and the fundamental for BIE could be found from Refs. [1–3]. Heritage and early history of the boundary element method was summarized more recently [4]. However, some difficult points for the BIE remain.

The degenerate scale problem in BIE is a particular boundary value problem in elasticity as well as in Laplace equation. It is known that the BIE in plane elasticity is generally formulated on the usage of the Somigliana's identity [3]. When the Dirichlet problem is formulated for the exterior domain, a BIE is obtained. In the Dirichlet problem for the exterior domain, the boundary displacements are the input data and the boundary tractions are the investigated arguments. If the assumed boundary displacements vanish, the BIE becomes a homogenous equation, or the right hand term in the equation is equal to zero. Such formulated homogenous BIE has been shown to yield a non-trivial solution for boundary tractions (or $\sigma_{ij}\neq 0$ at the boundary points), when the adopted configuration is equal to a degenerate scale. Since the displacement and stress condition at infinity, for example $u_i=O(\ln R)$, $\sigma_{ij}=O(1/R)$, or $u_i=O(1/R)$, $\sigma_{ij}=O(1/R^2)$, has not been defined exactly, it is expected to have a non-trivial solution from the viewpoint of mathematics and mechanics. This point will be discussed in detail later. Alternatively speaking, the relevant non-homogenous BIE has a non-unique solution when the degenerate scale is reached. From the viewpoint of engineering, the non-unique solution is an illogical one. Therefore, one must avoid meeting illogical solution caused by occurrence of the degenerate scale. Simply because the kernel from the displacement of fundamental solution does not depend on the normal of the boundary, the relevant homogenous equation for the Dirichlet problem for the interior domain must be the same style as that for the exterior domain. For the Laplace equation, the similar degenerate scale problem also exists.

The boundary integral equation is used for a ring region with the vanishing displacements along the boundary. In some particular scale of the configuration, the corresponding homogenous equation has non-trivial solution for the boundary tractions [5,6]. In fact, if the displacements are vanishing at the boundary of ring region, the stresses must also be equal to zero. Therefore, the obtained result seriously violates the basic property of elasticity.

Numerical technique for evaluating the degenerate scale was also suggested by using two sets of some particular solution in the geometry of normal scale [7].

Mathematical analysis of the degenerate scale problems for an elliptic-domain problem in elasticity was presented [8]. The analysis depends on the usage of the Airy stress function,

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0955-7997/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2008.05.007

which in turn is expressed through two complex potentials. By using the complex variable and the elliptic coordinate, a closed form solution for the degenerate scale was obtained. Both components of the displacement vanish for the boundary distribution when the degenerate scale occurs [8].

It is proved that there are either two single critical values or one double critical value for any domain boundary. For two circle holes in an infinite plate, degenerate scale was also studied [9].

Numerical procedure was developed to evaluate the degenerate scale directly from the zero value of a determinant. In the time of evaluating the eigenvalue, or the degenerate scale, the eigenfunction was also obtained as well [10,11]. In those papers, the influence for the used scale to the obtained solution is examined numerically. It is assumed that λ is the degenerate scale (or critical value) for size a. If the input data for the displacements are from an exact solution in closed form, and one chooses the $a=\lambda-\varepsilon$, $a=\lambda$ or $a=\lambda+\varepsilon$ ($\varepsilon=0.00002$ a small value), the obtained tractions along the boundary are seriously deviated from exact solution in closed form. In addition, if one chooses $a=0.9\lambda$ or $a=1.1\lambda$, small deviation in the computation is found.

It is found in this paper that, if the conformal mapping function for a notch configuration is known beforehand, the relevant degenerate scale (or the eigenvalue, or the critical value) and the solution for displacements and stresses (or the eigenfunction) can be obtained in closed form. Clearly, the present study does not rely on the numerical computation.

It was pointed out that the influence matrix of the weakly singular kernel (logarithmic kernel) might be singular for the Dirichlet problem of Laplace equation when the geometry is reaching some special scale [12]. In this case, one may obtain a non-unique solution. The non-unique solution is not physically realizable. From the viewpoint of linear algebra, the problem also originates from rank deficiency in the influence matrices [12]. By using a particular solution in the normal scale, the degenerate scale can be evaluated numerically [12]. The degenerate scale problem in BIE for the two-dimensional Laplace equation was studied by using degenerate kernels and Fourier Series [13]. In the circular domain case, the degenerate scale problem in BIE for the two-dimensional Laplace equation was studied by using degenerate kernels and circulants, where the circulants mean the influence matrices for the discrete system had a particular character [14].

This paper evaluates the degenerate scale for BIE in plane elasticity and antiplane elasticity by using the complex variable and the conformal mapping. Basic equations in the complex variable method of plane elasticity are compactly addressed. From the formulation of an exterior problem in plane elasticity, the background for existence of the degenerate scale is discussed in detail.

After using the conformal mapping, the vanishing displacement condition is assumed on the boundary of unit circle. The complex potentials on the mapping plane are sought in a form of superposition of the principal part and the complementary part. It is found from detail derivation that there are some complex potentials that satisfy the vanishing displacement condition along the boundary when the degenerate scale is reached. For four cases, the elliptic contour, the triangle contour, the square contour and the ellipse-like contour, the degenerate scales are evaluated. For the case of antiplane elasticity, similar degenerate scale problems are solved.

2. Preliminary knowledge

Basic equations in the complex variable method of plane elasticity are compactly addressed. From the formulation of an exterior problem in plane elasticity, the background for existence of the degenerate scale is discussed in detail. Formulation of the degenerate scale problem based on BIE is also introduced. Emphasis is on the usage of a new suggested kernel. Comparison between two techniques, from the usage of complex variable and the BIE, is stated in brief.

2.1. The representation form of the displacement–stress field in plane elasticity

The following analysis depends on the complex variable function method in plane elasticity [15]. In the method, the stresses $(\sigma_x, \sigma_y, \sigma_{xy})$, the resultant forces (X, Y) and the displacements (u, v) are expressed in terms of two complex potentials $\phi_1(z)$ and $\psi_1(z)$ such that

$$\sigma_x+\sigma_y = 4\,Re\,\phi_1'(z),$$
$$\sigma_y-\sigma_x+2i\sigma_{xy} = 2[\bar{z}\phi_1''(z)+\psi_1'(z)], \tag{1}$$

$$f=-Y+iX=\phi_1(z)+z\overline{\phi_1'(z)}+\overline{\psi_1(z)}, \tag{2}$$

$$2G(u+iv)=\kappa\phi_1(z)-z\overline{\phi_1'(z)}-\overline{\psi_1(z)}, \tag{3}$$

where $z=x+iy$ denotes complex variable, G is the shear modulus of elasticity, $\kappa=(3-v)/(1+v)$ is for the plane stress problems, $\kappa=3-4v$ is for the plane strain problems, and v is the Poisson's ratio. In the present study, the plane strain condition is assumed thoroughly. In the following, we occasionally rewrite the displacements "u", "v" as u_1, u_2, σ_x, σ_y, σ_{xy} as σ_{11}, σ_{22}, σ_{12}, and "x", "y" as x_1, x_2, respectively.

For any notch, for example, the elliptic notch in an infinite plate with resultant forces P_x and P_y applied on the contour and vanishing remote tractions (Fig. 1(a)), the relevant complex potentials can be expressed in the following form [15]:

$$\phi_1(z)=A_2\ \ln z+a_0+\sum_{k=1}^{\infty}\frac{a_k}{z^k},$$
$$\psi_1(z)=B_2\ \ln z+b_0+\sum_{k=1}^{\infty}\frac{b_k}{z^k}, \tag{4}$$

where

$$B_2=-\kappa\bar{A}_2,\quad A_2=-\frac{P_x+iP_y}{2\pi(\kappa+1)}. \tag{5}$$

The coefficients a_k and b_k ($k=1, 2, \ldots$) cane be evaluated from the solution of the boundary value problem, and two constants a_0 and b_0 represent the rigid translation of the body. Since two constants a_0 and b_0 are involved in Eq. (4), the complex potentials shown by Eq. (4) are expressed in an impure deformable form. Simply deleting two constants a_0 and b_0 in Eq. (4), we have:

$$\phi_1(z)=A_2\ \ln z+\sum_{k=1}^{\infty}\frac{a_k}{z^k},\quad \psi_1(z)=B_2\ \ln z+\sum_{k=1}^{\infty}\frac{b_k}{z^k}. \tag{6}$$

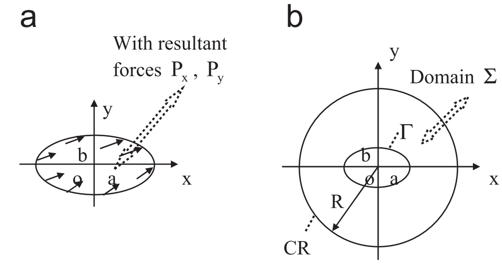

Fig. 1. (a) An elliptic hole in infinite plate with non-equilibrated loadings on the contour. (b) A ring-shaped domain Σ bounded by an elliptic contour Γ and a large circle "CR".

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