Engineering Analysis with Boundary Elements 33 (2009) 184–190

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

# On the necessity of non-conforming algorithms for 'small displacement' contact problems and conforming discretizations by BEM

A. Blázquez *, F. París

*Group of Elasticity and Strength of Materials, School of Engineering, University of Seville, Camino de los Descubrimientos s/n, Seville E-41092, Spain*

ARTICLE INFO

*Article history:*
Received 7 December 2007
Accepted 9 May 2008
Available online 2 July 2008

*Keywords:*
Contact problem
Conforming discretizations
Non-conforming discretizations
Weak approach

ABSTRACT

It has long been established that conforming algorithm approaches in BEM are reliable and robust for solving contact problems behaving under the hypothesis of small displacements and in the presence of initially conforming meshes. Non-conforming algorithm approaches have traditionally been associated with the presence of non-conforming meshes or with initially conforming meshes but in the presence of large displacements that alter the conformity of the meshes. In this paper, studying the Iosipescu test used to determine shear properties in composite materials, it has been shown that even in the presence of initial conforming meshes under the hypothesis of small displacements, the use of conforming algorithms leads to non-acceptable results, with incompatibilities in the displacement solution and with peaks in the distribution of the contact stresses. Several options to define the direction in which to establish the contact conditions were tested, all of them presenting problems. By contrast, the use of a non-conforming algorithm based on a weak application of the contact conditions previously developed by the authors reduces the incompatibilities and produces a smooth distribution of the contact stresses.


## 1. Introduction

Contact problems appear extensively in many engineering fields because loads are, in fact, usually applied or transmitted through contact between solids. The solution of contact problems is in general terms very difficult, and finding an analytical solution is only possible for very simple configurations [1].

The presence of non-simple geometries and/or loads requires the use of numerical methods. The first algorithms developed established contact conditions between pairs of nodes. References to Chan and Tuba [2] and Fredriksson [3] for FEM and to Andersson et al. [4] and París and Garrido [5] for BEM are classical, among others. In these algorithms the meshes of solids in contact were identical, which limited the scope of the analysis to the case of small displacements. The contact conditions were established between corresponding nodes, which formed a pair of nodes in contact along the whole application of the load.

There are situations where the use of conforming meshes for the bodies in contact is not an acceptable strategy of solution. There are three situations in which one uses non-conforming discretizations on the surfaces of the bodies that can have contact. The first is to facilitate the discretization work in contact problems between surfaces with different curvatures. The second is to reduce the size of the matrices involved in the problem when one of the bodies can be considered a rigid one, which allows the number of elements required to model it to be drastically reduced. Finally, the third is to properly solve problems that are geometrically non-linear due to the presence of large displacements. Different proposals have been made for algorithms allowing contact between surfaces with different meshes to take place. Thus, the works of Bathe and Chaudhary [6] and Klarbring and Björkman [7] for FEM, and Blázquez et al. [8], Olukoko et al. [9] and París et al. [10] for BEM, can be mentioned among others.

With reference to BEM, the initial approach for developing an algorithm to deal with non-conforming discretization (the three aforementioned papers belong to this class) was based on a strong application of the contact conditions. This approach could be considered a replica of the node-to-node contact scheme but allowing a node of one body to contact with any point of the discretized boundary of the other body. Blázquez et al. [11] showed that this scheme may produce some disturbance in the results and concluded some practical rules to avoid or diminish the appearance of these disturbances.

In subsequent papers Blázquez et al. [12–14] and Graciani et al. [15] developed a new approach based on a weak application of the contact conditions. This approach was proved to solve all the problems that had appeared using the strong approach of application of the contact conditions.

It will be shown in this paper that the use of a non-conforming algorithm (one based on the weak application of the contact

* Corresponding author. Tel.: +34 954 487299; fax: +34 954461637.
*E-mail address:* abg@esi.us.es (A. Blázquez).

0955-7997/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2008.05.003

conditions for the reasons formerly stated will be used) can be required, even in the absence of the three situations formerly invoked to require one such algorithm. In other words, a simple problem from the contact nature point of view that does not belong to any of the three cases mentioned and that consequently could be treated with conforming meshes, and that could even be solved with a node-to-node conforming algorithm, requires, if it is to be properly solved, not only a non-conforming approach but also an updating of the geometry. All this in presence of small displacements.

The next section is devoted to summarizing the contact algorithm used in this work, the attention being focused on the non-conforming approach. Section 3 describes the problem selected and shows the numerical results obtained. Finally, Section 4 presents the conclusions.

## 2. The contact problem

For the sake of brevity only a concise presentation of the features of the contact problem will be carried out, attempting basically to frame the problem studied. An extensive presentation of the problem can be found in [12].

### 2.1. Contact conditions

The frictionless case is a reversible problem (although not necessarily linear) that can be solved in one increment of load. Nevertheless, the equations to obtain the evolution of the variables will be presented in what follows in an incremental way.

Contact conditions have to be checked at points belonging to the area initially considered candidate for contact. This area is formed, for a certain value of the applied load, by two zones: a zone where contact has taken place but where there are points that could abandon it, and a zone that is free of contact but where there are points that could come into contact. Contact conditions are expressed, for both zones, in a local system defined by the normal and tangential directions. In 2D frictionless cases, 1 is the normal direction (outward to the body) and 2 the tangential direction (anticlockwise), see Fig. 1. The conditions that must be imposed in the zone in contact are

$$\Delta u_1^A + \Delta u_1^B = 0, \quad \Delta t_1^A = \Delta t_1^B, \quad \Delta t_2^A = \Delta t_2^B = 0 \tag{1}$$

In the zone free of contact, the components of the stress vector associated with the outward normal are known, usually taking zero value.

A point will be in contact as long as its normal stress component is in compression. A point will be free of contact as long as it does not interpenetrate the other solid.

Only the frictionless case, for the sake of brevity, is described in detail in this paper. The case corresponding to the presence of friction can be found in [5] for the conforming approach, and in [10] for the non-conforming approach.

### 2.2. Conforming algorithm for contact problems

When discretizations are initially conforming, a conforming algorithm can be used. In these cases, Eqs. (1) are imposed between pairs of nodes, each one (M and N in Fig. 1b) belonging to a body, and both defining a contact pair. In this way, with reference to the contact zone, the number of unknowns equals the number of equations [5].

### 2.3. Non-conforming algorithm for contact problems

When discretizations are non-conforming, the most straightforward approach is to extrapolate the idea of contact node pair that arises in conforming algorithms, then defining a contact pair between a node of one of the bodies and an internal point of one boundary element of the other body. Not all Eqs. (1) can be imposed on all these contact point pairs, because an over-constrained system (more equations than unknowns) would be obtained [8]. Some solutions were developed to avoid this difficulty, see [10] for a revision of procedures based on this approach. The basic idea of these procedures is to impose equilibrium conditions at points belonging to one of the solids (the mesh of the other solid will define traction distribution) and to impose compatibility conditions on the points of the other solid (the mesh of the first solid will then define displacement distribution). Approaches based on this scheme are called node-to-point or strong approaches. Blázquez et al. [11] showed that these algorithms could present some problems if the mesh used to describe displacement is not fine enough compared with the mesh defining tractions. In fact, it has to be much more fine, in the order of 5 to 10 times finer, depending on the problem. This does not imply a limitation of the procedure, but is simply an indication to assign the finer mesh to control the displacements.

An alternative approach, presented in [12], to the node-to-point approach is a weak application of the contact conditions. Equilibrium conditions are also imposed on one of the solids and compatibility conditions on the other, although now they are imposed in a weak sense using the Principle of Virtual Work. With this approach, similar meshes can be used in both solids to control

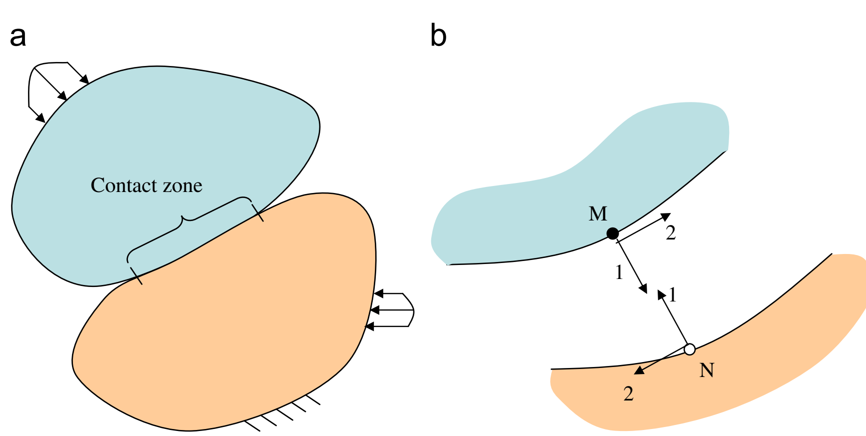


**Fig. 1.** The two-dimensional contact problem: (a) contact problem scheme and (b) local system for the contact equations.