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A family of hole boundary elements for modeling materials with cylindrical voids

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Abstract

In this paper, a boundary element formulation for modeling two-dimensional microstructures containing cylindrical voids is presented. Each void is inserted in a standard boundary element mesh in a efficient way using a single special *hole element*. The traditional discretization in several boundary elements is then avoided. Trigonometric functions are used as base for the element shape functions. A family of hole elements with 3, 4, 5 and 6 nodes is proposed. The numerical integration of the strongly singular kernels found in the element implementation is accomplished by the direct method, resulting in a regularized element. The convergence behavior of the proposed approach is analyzed for several quadrature orders. A static condensation scheme is performed on the system of equation to further improve the formulation. The accuracy of proposed method is illustrated with some examples.

Keywords: Boundary element method; Microporous material; Singular integrals; Integral regularization

1. Introduction

The determination of the macroscopic properties of heterogeneous materials is an essential topic of engineering and materials science. The study of relationships between microstructural phenomenon and the macroscopic behavior not only allows to predict the behavior of existing materials, but also provides a tool for the design of microstructures such that the resulting macroscopic behavior agrees with *a priori* specified characteristics. Several analytical and semi-analytical methods have been proposed to perform these predictions, such as bounds and direct methods (see, for instance, Refs. [1,2]). In addition, one could mention asymptotic models which are either of dilute (for example, two-phase composites with small inclusions) or densely packed (with defects that may be close to touching) types, as well as high-order multi-pole approximations [3,4]. The combination of micro-mechanics and numerical methods supplies a powerful tool for material behavior modeling. However, the use of domain methods like the finite element method (FEM) is not very attractive in these cases since it requires the generation of domain meshes (Fig. 1a). This aspect becomes more conspicuous as the number of heterogeneities in the material sample becomes larger. Moreover, the design of new materials with randomly distributed inhomogeneities requires many simulations of different samples with the same volume fraction, creating a serious computational overhead related to mesh generation issues. The boundary element method (BEM) has already shown to be accurate and efficient for many problems in solids mechanics. Particularly, in overall properties predictions, the BEM can deliver very efficient predictions [5] due to the fact that spatial average schemes of the internal fields only require information on the microstructure boundary. Therefore, the boundary-only discretization nature of the BEM becomes a very attractive advantage (Fig. 1b). In fact, a number of boundary methods have been developed and implemented to predict the behavior of composite microstructures during the last years [6–14].

The aim of this work is to develop and extend an even more economical BEM approach to analyze solids

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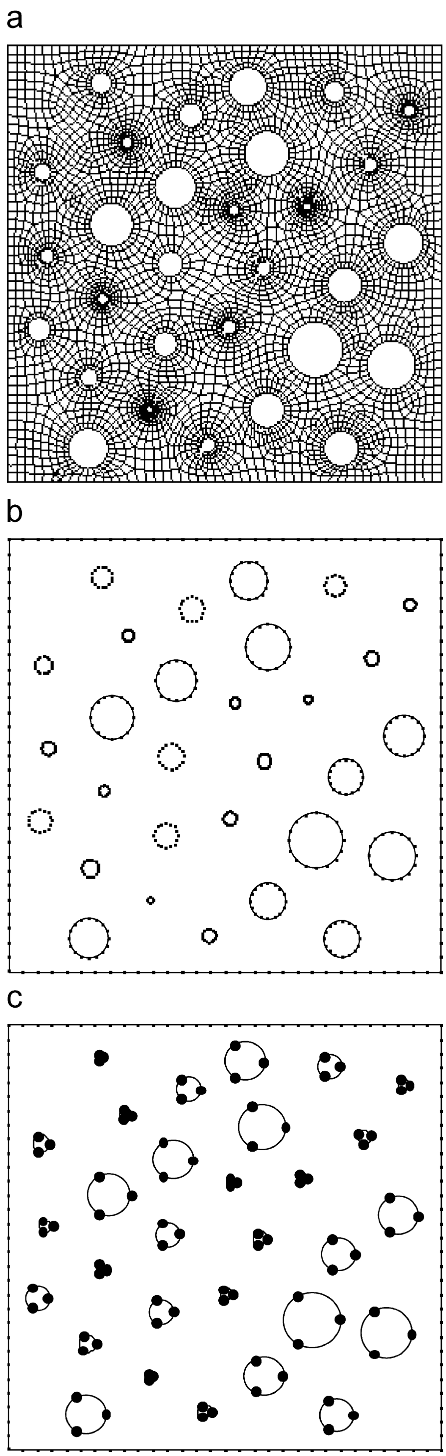

Fig. 1. Typical discretizations of a porous material sample: (a) FEM; (b) conventional BEM; (c) BEM with hole elements.

containing circular cavities. The formulation of Henry and Banerjee [15] for modeling tubular holes is particularized for two dimensions. Each cavity is modeled with a single, specially developed hole element, reducing both the amount of data and computational cost of the conventional BEM formulation (Fig. 1c). Trigonometric functions are proposed as a base for the element shape functions resulting in elements with 4, 5 and 6 nodes in addition to original element of 3 nodes proposed in Ref. [15]. The material is considered as an isotropic and homogeneous matrix containing cylindrical holes.

The numerical integration of strongly singular kernels plays a key role in the implementation of many integral equation methods. In the last two decades, several methodologies have been proposed to perform the task. Nevertheless, only a few of them have generality for use with general fundamental tensors and higher order element shape functions. The direct method seems to be one of the most general since it imposes no formal restriction on the type of kernel to be integrated and enables the use of standard Gaussian quadrature rules [16–18]. On the other hand, this method requires the knowledge of the analytical asymptotic expansions of the kernels around the singular pole. In this work, these expansions are derived in order to regularize the strongly singular integrals present in the hole element.

The resulting element family is useful for design and optimization analysis of mechanical components containing many holes, but it is particularly suitable for modeling and designing microporous material with random distribution of the inhomogeneities, when the evaluation of effective properties is desired.

2. Boundary element formulation

2.1. Boundary integral formulation for a matrix with cylindrical holes

The direct boundary integral equation for displacements [19] of an elastic solid can be applied to solve the Boundary Value Problem (BVP) of a domain Ω surrounded by an exterior boundary Γ_o containing cylindrical micro-holes with boundary Γ. In the absence of body forces the equation is expressed as [15]:

$$\mathbf{C}_{ij}(\xi)u_i(\xi) = \int_{\Gamma_o} [G^o_{ij}(x,\xi)t^o_i(x) - F^o_{ij}(x,\xi)u^o_i(x)]\,\mathrm{d}\Gamma + \sum_{n=1}^{N_f} \int_{\Gamma^n} [G^f_{ij}(x,\xi)t^f_i(x) - F^f_{ij}(x,\xi)u^f_i(x)]\,\mathrm{d}\Gamma, \tag{1}$$

where i,j denote Cartesian components, u_i and t_i are the boundary displacement and tractions, G_{ij} and F_{ij} are the Kelvin fundamental solution at a point ξ (field point) due to the unit load located at x (load point), $\mathbf{C}_{ij}(\xi)$ is a function of boundary geometry at ξ and N_f is the number of holes. The superscripts o and f (no sum) refer to the quantities on the outer boundary of the matrix and on the holes boundaries, respectively. Contrary to the conventional discretization of Eq. (1), which requires fine meshing around the each hole, the present approach allows an efficient analysis, significantly reducing the input data amount and the total number of degree of freedom.