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Radiation and resonant frequency of a resistive patch and uniaxial anisotropic substrate with entire domain and roof top functions

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Abstract

This paper develops and describes briefly the spectral domain analysis of microstrip patch antennas in which two sets of basis functions are used to model the patch current: the entire domain sinusoid basis functions and roof-top sub-domain basis functions. The moment method technique based on Galerkin's procedure is developed to examine the resonant frequency of a perfectly or an imperfectly conducting microstrip rectangular patch, which is printed on isotropic or uniaxial anisotropic substrate. It is seen that the results obtained by the roof top basis functions provides a significant improvement in the computations time with less iterations in the evaluation of the resonant frequency of a microstrip patch compared with the entire domain basis functions. Numerical results for the effects of surface resistance and uniaxial substrate on the radiation and the directivity of the rectangular microstrip structure are also presented. The accuracy of the computed technique is presented and compared with other computed results.

Keywords: Rectangular; Resistive patch; Uniaxial; Entire domain; Roof top functions

1. Introduction

Several basic methods of numerical analysis have been described in the literature for solving electromagnetic problems. Among these we can cite: the finite-difference method (FDM), the finite-element method (FEM), the transmission line matrix (TLM) method, the boundary element method (BEM) and the moment method (MoM). The most important criteria in selecting methods are the rapidity and effort to derive the algorithm and write the program, storage requirement and computer run time necessary to obtain desired accuracy [1]. The full-wave moment method which achieves greater computational efficiency [1] has been applied extensively and is now a standard approach for analysis of microstrip geometry. Many authors [2] have used entire domain expansion functions; the majority of work has examined the input impedance and scattering properties of perfectly conducting patches. The boundary condition for the electric field on a thin resistive sheet has been studied by Senior [3] and is valid as long as the sheet is electrically thin. Using this type of boundary condition, several authors have studied the scattering response of resistive strips, tapered resistive strips and frequency selective surfaces [4,5]. In this paper, the integral equation is formulated using the Green's function on isotropic or uniaxial anisotropic dielectric substrate to determine the electric field at any point. The solution of such an integral equation is obtained by the moment method with the given set of boundary conditions. From this analysis, the current distribution on the patch is determined. One of the main problems with the computational procedure is to overcome the complicated time-consuming task of calculating the Green's functions in the procedure of resolution by the moment method. Clearly, the choice of the basis functions used to calculate the current of a rectangular patch is crucial for obtaining numerical convergence with faster times. Fast numerical convergence is obtained using subdomain roof top basis functions to expand the current

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on the patch with a simplified algebraic formulation. Also the study of anisotropic substrate materials is interesting since it has been found that the use of such materials may have a beneficial effect on circuit or antenna [2]. However, the designers should carefully check for the anisotropic effects in the substrate materials with which they will work. Since the effect of non-zero surface resistance on the scattering properties of a microstrip antenna on a uniaxial substrate has not yet been treated [5], only for a few works have incorporated this effect on an isotropic substrate [3,6], a number of results pertaining to the case of uniaxial substrate are presented in this paper.

2. Theory

The patch is assumed to be located on a grounded dielectric slab of infinite extent. The resistive rectangular patch with length a and width b is embedded in a single substrate containing isotropic or anisotropic materials with the optical axis normal to the patch and has a uniform thickness h (Fig. 1), the permittivity tensor in the case of uniaxial anisotropic substrate is given in Refs. [2,5,7]. All the electric materials are assumed to be non-magnetic with permeability μ_0. To simplify the analysis, the antenna feed will not be considered.

The boundary condition on the patch is given by [3,5]

$$\mathbf{E}_{scat} + \mathbf{E}_{inc} = \mathbf{R}_s \cdot \mathbf{J}, \tag{1}$$

where $\mathbf{E}_{scat}$, $\mathbf{E}_{inc}$ and $\mathbf{J}$ are, respectively, the tangential components of scattered electric field, the tangential components of incident electric field and the surface current on the patch. The surface resistance $\mathbf{R}_s$ is in general a function of x and y and is equal to zero for a perfectly conducting patch.

From Maxwell's equations in the Fourier transform domain, knowing the general form of the longitudinal components $\tilde{\mathbf{E}}_z$ and $\tilde{\mathbf{H}}_z$, the transverse fields in the (TM, TE) representation can be written in terms of these longitudinal components as [7]

$$\tilde{\mathbf{E}}_s(\mathbf{k}_s, z) = \begin{bmatrix} \tilde{\mathbf{E}}_s^{TM}(\mathbf{k}_s, z) \\ \tilde{\mathbf{E}}_s^{TE}(\mathbf{k}_s, z) \end{bmatrix} = e^{i\bar{\mathbf{k}}_z z}\mathbf{A}(\mathbf{k}_s) + e^{-i\bar{\mathbf{k}}_z z}\mathbf{B}(\mathbf{k}_s), \tag{2}$$

$$\tilde{\mathbf{H}}_s(\mathbf{k}_s, z) = \begin{bmatrix} \tilde{\mathbf{H}}_s^{TM}(\mathbf{k}_s, z) \\ \tilde{\mathbf{H}}_s^{TE}(\mathbf{k}_s, z) \end{bmatrix} = \bar{\mathbf{g}}(\mathbf{k}_s)\left[e^{i\bar{\mathbf{k}}_z z}\mathbf{A}(\mathbf{k}_s) - e^{-i\bar{\mathbf{k}}_z z}\mathbf{B}(\mathbf{k}_s)\right], \tag{3}$$

where $\mathbf{A}$ and $\mathbf{B}$ are two unknown vectors to be determined [7], $\mathbf{k}_s$ is the transverse wave vector and $\bar{\mathbf{g}}(\mathbf{k}_s)$ is determined by

$$\begin{bmatrix} \frac{\omega\varepsilon_0\varepsilon_x}{k_z^{TM}} & 0 \\ 0 & \frac{k_z^{TE}}{\omega\mu_0} \end{bmatrix}. \tag{4}$$

$$\bar{\mathbf{k}}_z = \begin{bmatrix} k_z^{TM} & 0 \\ 0 & k_z^{TE} \end{bmatrix}, \quad k_z^{TM} = \left(\varepsilon_x k_0^2 - \frac{\varepsilon_x}{\varepsilon_z}k_s^2\right)^{1/2},$$

$$k_z^{TE} = (\varepsilon_x k_0^2 - k_s^2)^{1/2}, \quad k_0 = \omega\sqrt{\varepsilon_0\mu_0},$$

where k_z^{TM} and k_z^{TE} are, respectively, the propagation constants for TM and TE waves in the uniaxial dielectric; ω, μ_0 and ε_0 are the angular frequency, permeability and permittivity of free space, respectively.

ε_z is the permittivity along the optical axis z and ε_x is the permittivity along the axis x.

By eliminating the unknowns $\mathbf{A}$ and $\mathbf{B}$ in Eqs. (2) and (3), we obtain the following matrix which combines the tangential field components on both sides of the considered layer (Fig. 1) as input and output quantities:

$$\begin{bmatrix} \tilde{\mathbf{E}}^{TM(TE)}(\mathbf{k}_s, z_2^-) \\ \tilde{\mathbf{H}}^{TM(TE)}(\mathbf{k}_s, z_2^-) \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{I}}\cos(k_z^{TM(TE)}h) & i\bar{\mathbf{g}}^{-1}\sin(k_z^{TM(TE)}h) \\ i\bar{\mathbf{g}}\sin(k_z^{TM(TE)}h) & \bar{\mathbf{I}}\cos(k_z^{TM(TE)}h) \end{bmatrix} \times \begin{bmatrix} \tilde{\mathbf{E}}^{TM(TE)}(\mathbf{k}_s, z_1^+) \\ \tilde{\mathbf{H}}^{TM(TE)}(\mathbf{k}_s, z_1^+) \end{bmatrix} - \begin{bmatrix} 0 \\ \tilde{\mathbf{J}}^{TM(TE)}(\mathbf{k}_s) \end{bmatrix}, \tag{5}$$

where $\bar{\mathbf{I}}$ is the unit matrix.

In the frequency domain, the relationship between the patch current and the electric field on the patch is given by

$$\tilde{\mathbf{E}}_s(\mathbf{k}_s) = \bar{\mathbf{G}}(\mathbf{k}_s) \cdot \tilde{\mathbf{J}}(\mathbf{k}_s), \tag{6}$$

where $\bar{\mathbf{G}}$ is the spectral dyadic Green's function which is efficiently determined by Eq. (7) and $\tilde{\mathbf{J}}(\mathbf{k}_s)$ is the current on the patch which relates to the vector Fourier

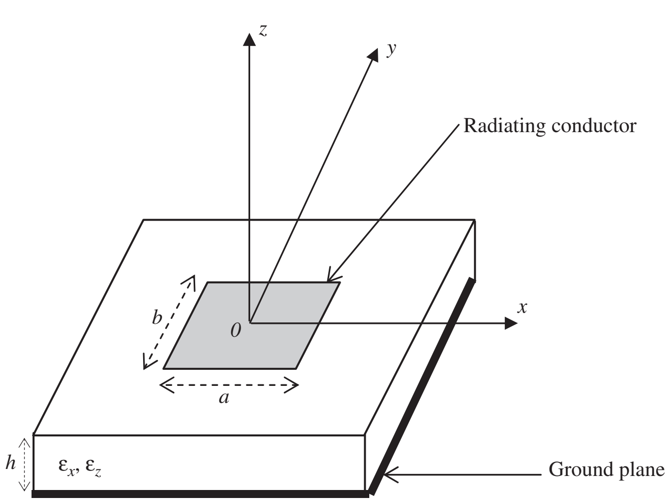

Fig. 1. Geometry of a rectangular microstrip antenna.