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A three-dimensional acoustics model using the method of fundamental solutions

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Abstract

The method of fundamental solutions (MFS) is formulated in the frequency domain to model the sound wave propagation in three-dimensional (3D) enclosed acoustic spaces. In this model the solution is obtained by approximation, using a linear combination of fundamental solutions for the 3D Helmholtz equation. Those solutions relate to a set of virtual sources placed over a surface placed outside the domain in order to avoid singularities. The materials coating the enclosed space surfaces can be assumed to be sound absorbent. This effect is introduced in the model by imposing impedance boundary conditions, with the impedance being defined as a function of the absorption coefficient. To impose these boundary conditions, a set of collocation points (observation points) needs to be selected along the boundary.

Time domain responses are obtained by applying an inverse Fourier transform to the former frequency domain results. In order to avoid "aliasing" phenomena in the time domain results, the computations introduce damping in the imaginary part of the frequency. This effect is later removed in the time domain by rescaling the response.

After corroborating the present solution against the analytical solution, known in closed form for the case of a parallelepiped room bounded by rigid walls, the model is used to solve the case of a dome.

Keywords: MFS; Meshless; Acoustic model; Three-dimensional model

1. Introduction

The performance of rooms used for speech or music is highly influenced by the correct choice of a set of parameters during the design stage. The sound field produced inside enclosed spaces is dependent on their volume, geometry, coating materials, sound frequency, and occupancy. For this reason, the acoustics of rooms has been researched for many years in order to obtain models and experimental results that will be helpful to acoustic design. The modeling of the phenomena involved is not simple and different numerical methods of varying complexity have been developed.

There are classic statistical models, following the well-known Sabine and Eyring theories that consider uniform energy density distribution, and recently some statistical models have been improved to include non-uniform reverberating energy density distribution [1].

Methods based on geometric acoustics are also widely used in room acoustics prediction. Among these methods is the image source method [2,3] where the huge number of virtual sources required can be a limitation, and the ray tracing technique [4], valid in the high frequency range but including a degree of uncertainty since it is not sure that all the rays needed are included in the response. There are also hybrid methods combining those two [5].

Methods requiring domain discretization such as the finite element method (FEM), the finite difference method, and the boundary element method (BEM), have not been widely used to compute the propagation of sound, because of the high computation cost entailed. The FEM [6] and the finite difference method [7] fail because the domain under consideration has to be fully discretized, and very fine meshes are needed to solve excitations at high frequencies. Methods like the BEM [8] are more efficient

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in terms of computer cost as they only require the discretization of the boundaries, but they involve a large computational effort, particularly for very high frequencies.

Recently, several researchers have focused their work on meshless methods in order to avoid the time-consuming problem originated in mesh generation for complicated geometries. These methods have been used to solve some acoustic problems. The method of fundamental solutions (MFS) is applicable when a fundamental solution of the differential equation in question is known. Recent survey papers describe the method and various applications for it [9–11]. In Ref. [11], Fairweather et al. described and reviewed the MFS and related methods for the numerical solution of scattering and radiation problems in fluids and solids. Alves and Valtchev compared the plane waves method and the MFS for acoustic wave scattering [12]. Suleau and Bouillard applied the element-free Galerkin method to compute harmonic solutions of acoustic problems, governed by a Helmholtz equation [13]. Chen et al. employed the boundary collocation method using radial basis functions for the acoustic eigenanalysis of three-dimensional (3D) cavities [14]. In this paper, the MFS method is implemented to model a 3D acoustic problem. This method suffers from ill-conditioning of the system's linear equations, which is common when external source collocation methods are applied. Several techniques have been developed to handle the ill-conditioning of similar meshless collocation methods. Some use compactly supported radial basis functions [15], while others incorporate the least squares approach [16,17], apply a pre-conditioning technique [18] or use the matrix-free greedy algorithm [19].

Although the single value decomposition method (SVD) has traditionally been employed to solve ill-posed problems, in the case of the MFS, Chen et al. [20] demonstrated that the SVD is no more reliable than Gaussian elimination for non-noise boundary conditions. However, for noise boundary data, the truncated singular value decomposition method (TSVD) has been found to be more efficient than Gaussian elimination.

In this work, the sound field generated by a 3D sound source inside a 3D enclosure is modeled using the MFS. The model developed allows the boundaries to be rigid or absorbent and the final system of equations is solved using Gaussian elimination.

The problem is first formulated, the results are then validated using the image source method, and finally an application is presented.

2. Problem formulation

The pressure amplitude generated by a 3D source inside an air-filled 3D enclosed space is calculated by the MFS in the frequency domain (ω). The response inside the domain is found as a linear combination of fundamental solutions for the governing equation. Thus, the scattered pressure (p) wave field is written as

$$p = \sum_{s=1}^{N} [a_s G(\boldsymbol{x}, \boldsymbol{x_s}, \omega)]. \tag{1}$$

These solutions represent the sound field generated by a set of N virtual sources with amplitude a_s, placed outside the domain on a fictitious boundary in order to avoid singularities. $G(\boldsymbol{x}, \boldsymbol{x_s}, \omega)$ is the 3D Green's function for pressure, for a receiver placed at $\boldsymbol{x}$ with co-ordinates (x, y, z), generated by pressure sources located at $\boldsymbol{x_s}$ with co-ordinates (x_s, y_s, z_s).

The 3D Green's function for pressure is well known

$$G(x, x_s, \omega) = \frac{e^{-i(\omega/c)r}}{r}, \tag{2}$$

where $r = \sqrt{(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2}$, c is the sound wave velocity and $i = \sqrt{-1}$. The coefficients a_s are obtained by imposing the required boundary conditions at M collocation points (x_k, y_k, z_k) along the boundary. A system of M equations by N unknowns is then obtained. In this work, an equal number of collocation points and sources was considered, leading to a system $M \times M$. The resulting linear system was solved by Gaussian elimination [20].

For rigid enclosures, null velocities (incident velocity plus reflected velocity) are ascribed to the boundary. The Green's function for velocities is then given by

$$H(\boldsymbol{x_s}, \boldsymbol{x_k}, \omega, n) = -\frac{1}{i\rho\omega} \frac{\partial G(\boldsymbol{x}, \boldsymbol{x_s}, \omega)}{\partial r} \frac{\partial r}{\partial n}, \tag{3}$$

where ρ is the air density and n is the unit outward normal at the collocation point (x_k, y_k, z_k). When the room's coating material is absorbent the governing equation is given by

$$G(\boldsymbol{x_s}, \boldsymbol{x_k}, \omega) + \hat{Z} H(\boldsymbol{x_s}, \boldsymbol{x_k}, \omega, \boldsymbol{n}) = 0, \tag{4}$$

where $\hat{Z}$ is the material impedance given by the ratio between the pressure and velocity.

The material impedance can be expressed using the absorption coefficient α considering that $p_r = Rp_{inc}$, $v_r = -Rv_{inc}$, $R = \sqrt{1 - \alpha}$, where R is the reflection coefficient, p_r the reflected pressure, p_{inc} the incident pressure, v_r the reflected velocity, and v_{inc} is the incident velocity. In fact, $\hat{Z}$ can be expressed as

$$\hat{Z} = \frac{p_{inc} + Rp_{inc}}{v_{inc} + Rv_{inc}} = \frac{p_{inc}}{v_{inc}} \left(\frac{1 + \sqrt{1 - \alpha}}{1 - \sqrt{1 - \alpha}} \right), \tag{5}$$

where $p_{inc} = (e^{-i(\omega/c)r}/r)$ and $v_{inc} = -([-i(\omega/c)r - 1]e^{-i(\omega/c)r})/i\rho\omega r^2)(\partial r/\partial n)$.

In the case of an enclosure of arbitrary geometry built over a horizontal rigid base, the placement of collocation points at this surface can be avoided if an appropriate Green's function for a half-space is used:

$$G(\boldsymbol{x}, \boldsymbol{x_k}, \omega) = \frac{e^{-i(\omega/c)r}}{r} + \frac{e^{-i(\omega/c)r'}}{r'} \tag{6}$$