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Elastodynamics of a crack on the bimaterial interface

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ABSTRACT

The paper is an application of boundary integral equations to the problem of a crack located on the bimaterial interface under time-harmonic loading. A system of linear algebraic equations is derived for solving the problem numerically. The distributions of the displacements and tractions at the bimaterial interface are obtained and analysed for the case of a penny-shaped crack under normal tension-compression wave. The dynamic stress intensity factors (normal and shear modes) are also computed. The results are compared with those obtained for the static case.

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1. Introduction

Modern design and service conditions require a continued increase in the magnitude, velocity and frequency of the loading of various mechanical systems. The level of safety requirements increases consequently because the cost of unpredictable fracture is enormously high. Unfortunately, it is not possible to fully eliminate the appearance of different micro-defects in structural materials due to the processing flaws, fatigue, consequences of an impact, etc. In particular, localized damage in the form of distributed cracks/delaminations often occurs at the bonding surfaces between two dissimilar materials. The presence of cracks and delaminations considerably decreases the strength and the lifetime of structures as well as significantly increases the cost of exploitation. Cracks act as local stress concentrators, which can lead to a sudden fracture under unexpectedly small loading and, apart from the economic value, put human health and life at risk. Hence, the elastodynamic response of intra- and inter-component cracks to an elastic wave is a topic of long-standing interest in the study of wave propagation [1–5].

A considerable body of work is devoted to the solution of two- and three-dimensional fracture mechanics problems for cracked *homogeneous* solids under dynamic loading; see, for example, [1–9]. It is only possible to solve these problems using advanced numerical methods, since the analytical solutions are limited to a relatively small number of idealized model problems corresponding to the very special geometrical configurations and loading conditions. Among many numerical methods, which are available in the literature, the boundary integral equations method provides a powerful and efficient tool for the problems of wave propagation in cracked solids, due to its semi-analytical nature, reduced problem dimension, high accuracy and easy processing of input and output data [3–5,8,10–15].

One of the major difficulties encountered when using the boundary integral equations in elastodynamic problems is the evaluation of divergent integrals with different types of singularity. The order of the singularity and the structure of integrals depend on the type of weight functions. These integrals are often hyper-singular and should be treated in the sense of the Hadamard finite part [16–18].

Elastodynamic investigation of *interface* cracks received considerably less attention than the case of cracks in homogeneous materials due to the substantial complications, which arise in numerical solution of such problems. The recent papers [19,20] derive the system of boundary integral equations for the three-dimensional problem for an interface crack between two dissimilar elastic materials under general dynamic loading. For the case of harmonic loading, the required integral kernels, which are fundamental solutions of the elastodynamics in the frequency domain, were given in [21]. In Refs. [20–22], the distributions of displacements and tractions at the bonding interface and the crack surface were computed for the case of a penny-shaped crack under normally incident tension-compression wave for several typical material properties of half-spaces. It was shown that with decreasing frequency of the loading the dynamic solution tends to the static one, and the obtained numerical results are in a very good agreement with the analytical static solution [23,24] and numerical static solution [18].

In this paper for the first time in the case for an interface crack under harmonic loading, the dynamic stress intensity factors

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(opening and transverse shear modes) are computed and their distribution with respect to the dimensionless wave number is analysed. The collocation method with a piecewise-constant approximation is used for solving the problem numerically. The resulting system of linear algebraic equations is given and its structure is analysed. It is shown that the displacements and tractions at the bonding interface decrease gradually with increase in the distance to the crack, i.e. the Sommerfeld radiation-type condition is satisfied at the infinity.

2. Methodology

Let us consider a planar crack located at the bimaterial interface. For this purpose, we investigate an unbounded three-dimensional elastic solid, which consists of two homogeneous isotropic half-spaces. The interface between the half-spaces, Γ^*, acts as the boundary $\Gamma^{(1)}$ for the upper half-space, and the boundary $\Gamma^{(2)}$ for the lower half-space. The planes $\Gamma^{(1)}$ and $\Gamma^{(2)}$ differ by the opposite orientation of their outer normal vectors. The material is undergoing a harmonic loading with the frequency $\omega = 2\pi/T$.

We denote the vector of displacements $\mathbf{u}^{(m)}(\mathbf{x},t)$ and the traction vector $\mathbf{p}^{(m)}(\mathbf{x},t)$. Henceforth, superscript (1) refers to the upper half-space and superscript (2) refers to the lower half-space. We assume that the surface $\Gamma^{(m)}$ ($m = 1{,}2$) consist of the infinite part $\Gamma^{(m)*}$ and the finite part $\Gamma^{(m)\mathrm{cr}}$. The crack surface Γ^{cr} is formed by two faces, $\Gamma^{(1)\mathrm{cr}}$ and $\Gamma^{(2)\mathrm{cr}}$, i.e. $\Gamma^{(m)} = \Gamma^{(m),*} \cup \Gamma^{(m),\mathrm{cr}}$ and $\Gamma^{\mathrm{cr}} = \Gamma^{(1)\mathrm{cr}} \cup \Gamma^{(2)\mathrm{cr}}$ (see Fig. 1).

The conditions of continuity for displacements and stresses are satisfied at the interface $\Gamma^* = \Gamma^{(1)} \cap \Gamma^{(2)}$. In the absence of body forces, the stress–strain state of both domains is defined by the Lamé dynamic equations of the linear elasticity, and the Sommerfeld radiation-type condition is imposed at infinity on the vector of displacements.

The procedure for deriving the system of boundary integral equations is loosely based on the original ideas put forward in [11–14]. The detailed description is given by the authors in [19,20]. It was shown that using the Somigliana dynamic identity, the following system of boundary integral equations for displacements and tractions at the interface and the crack faces can be obtained:

$$\frac{1}{2}g_j^{(m)}(\mathbf{x},t) - \int_T \int_{\Gamma^{(m)\mathrm{cr}}} g_i^{(m)}(\mathbf{y},\tau)K_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$= -\int_T \int_{\Gamma^{(m)\mathrm{cr}}} u_i^{(m)}(\mathbf{y},\tau)F_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$-\int_T \int_{\Gamma^*} p_i^*(\mathbf{y},\tau)K_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$+\int_T \int_{\Gamma^*} u_i^*(\mathbf{y},\tau)F_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau, \quad \mathbf{x}\in\Gamma^{(m)\mathrm{cr}}, \quad m=1,2, \tag{1}$$

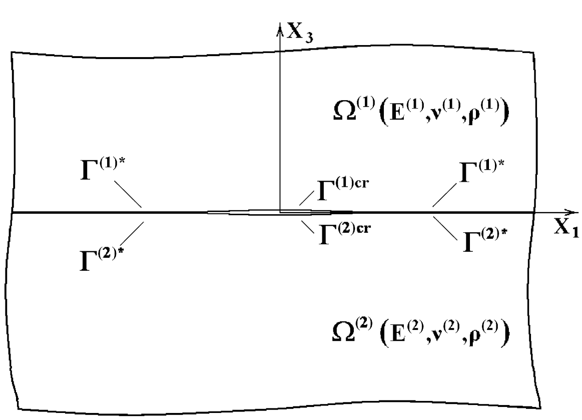

Fig. 1. Interface crack between two dissimilar half-spaces.

$$\int_T \int_{\Gamma^{(1)\mathrm{cr}}} g_i^{(1)}(\mathbf{y},\tau)K_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$-\int_T \int_{\Gamma^{(2)\mathrm{cr}}} g_i^{(2)}(\mathbf{y},\tau)K_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$=\int_T \int_{\Gamma^{(1)\mathrm{cr}}} u_i^{(1)}(\mathbf{y},\tau)F_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$-\int_T \int_{\Gamma^{(2)\mathrm{cr}}} u_i^{(2)}(\mathbf{y},\tau)F_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$+\int_T \int_{\Gamma^*} p_i^*(\mathbf{y},\tau)(K_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)+K_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau))\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$-\int_T \int_{\Gamma^*} u_i^*(\mathbf{y},\tau)(F_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)+F_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau))\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau, \quad \mathbf{x}\in\Gamma^*, \tag{2}$$

$$\int_T \int_{\Gamma^{(1)\mathrm{cr}}} g_i^{(1)}(\mathbf{y},\tau)U_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$-\int_T \int_{\Gamma^{(2)\mathrm{cr}}} g_i^{(2)}(\mathbf{y},\tau)U_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$=\int_T \int_{\Gamma^{(1)\mathrm{cr}}} u_i^{(1)}(\mathbf{y},\tau)W_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$-\int_T \int_{\Gamma^{(2)\mathrm{cr}}} u_i^{(2)}(\mathbf{y},\tau)W_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau)\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$+\int_T \int_{\Gamma^*} p_i^*(\mathbf{y},\tau)(U_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)+U_{ij}^{(2)}(\mathbf{y},\mathbf{y},t-\tau))\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau$$
$$-\int_T \int_{\Gamma^*} u_i^*(\mathbf{y},\tau)(W_{ij}^{(1)}(\mathbf{x},\mathbf{y},t-\tau)+W_{ij}^{(2)}(\mathbf{x},\mathbf{y},t-\tau))\,\mathrm{d}\mathbf{y}\,\mathrm{d}\tau, \quad \mathbf{x}\in\Gamma^*, \tag{3}$$

where $\mathbf{g}^{(m)}(\mathbf{x},t), \mathbf{x}\in\Gamma^{(m)\mathrm{cr}} (m=1,2)$ are known traction vectors at the crack opposite faces, which are determined by the external load.

For the case of harmonic loading, which is considered in the paper, the components of the stress–strain state can be expressed as harmonic functions:

$$\mathbf{p}(\mathbf{x},t) = \mathrm{Re}\{\mathbf{p}(\mathbf{x})e^{i\omega t}\} = \mathbf{p}_c(\mathbf{x})\cos(\omega t) + \mathbf{p}_s(\mathbf{x})\sin(\omega t), \tag{4}$$

$$\mathbf{u}(\mathbf{x},t) = \mathrm{Re}\{\mathbf{u}(\mathbf{x})e^{i\omega t}\} = \mathbf{u}_c(\mathbf{x})\cos(\omega t) + \mathbf{u}_s(\mathbf{x})\sin(\omega t), \tag{5}$$

where

$$\mathbf{p}(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T \mathbf{p}(\mathbf{x},t)e^{-i\omega t}\,\mathrm{d}t, \quad \mathbf{u}(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T \mathbf{u}(\mathbf{x},t)e^{-i\omega t}\,\mathrm{d}t, \tag{6}$$

$$\mathbf{p}_c(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T \mathbf{p}(\mathbf{x},t)\cos(\omega t)\,\mathrm{d}t,$$
$$\mathbf{p}_s(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T \mathbf{p}(\mathbf{x},t)\sin(\omega t)\,\mathrm{d}t, \tag{7}$$

$$\mathbf{u}_c(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T \mathbf{u}(\mathbf{x},t)\cos(\omega t)\,\mathrm{d}t,$$
$$\mathbf{u}_s(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T \mathbf{u}(\mathbf{x},t)\sin(\omega t)\,\mathrm{d}t. \tag{8}$$

Then the system of boundary integral equations (1)–(3) can be re-written in the following form, which does not contain the time integrals:

$$\frac{1}{2}g_j^{(m)}(\mathbf{x}) - \int_{\Gamma^{(m)\mathrm{cr}}} g_i^{(m)}(\mathbf{y})K_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)\,\mathrm{d}\mathbf{y}$$
$$= -\int_{\Gamma^{(m)\mathrm{cr}}} u_i^{(m)}(\mathbf{y})F_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)\,\mathrm{d}\mathbf{y} - \int_{\Gamma^*} p_i^*(\mathbf{y})K_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)\,\mathrm{d}\mathbf{y}$$
$$+\int_{\Gamma^*} u_i^*(\mathbf{y})F_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)\,\mathrm{d}\mathbf{y}, \quad \mathbf{x}\in\Gamma^{(m)\mathrm{cr}}, \quad m=1,2, \tag{9}$$

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