Engineering Analysis with Boundary Elements 33 (2009) 302–313

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Local buckling of thin-walled structures by the boundary element method

P.M. Baiz *, M.H. Aliabadi

Imperial College London, South Kensington, London, UK

ARTICLE INFO

Article history:
Received 24 April 2008
Accepted 28 July 2008
Available online 23 September 2008

Keywords:
Assembled plates and shallow shells
Buckling
Shear deformation theory
Boundary element method

ABSTRACT

In this work a multi-region boundary element formulation for linear local buckling analysis of assembled plate and shallow shell structures is presented. The assembly is divided into sub-regions. In each sub-region, the formulation is formed by coupling boundary element formulations of shear deformable plate bending and two-dimensional plane stress elasticity. Domain integrals appearing in the formulation (due to the curvature and due to the domain load) are transformed into equivalent boundary integrals. Membrane stresses at discrete domain points of each sub-region (plate or shallow shell) in the assembly are obtained from the prebuckling state, resulting in a set of linear buckling equations in terms of the buckling deflection and the buckling load factor. Buckling equation is presented as a standard eigenvalue problem. Results are compared with FEM solutions and it is shown that good accuracy can be achieved with the present multi-region BEM formulation.

1. Introduction

Structures made from plate and shell assemblies, also known as thin-walled structures, are important in many areas of engineering. Their advantage relies on their principal characteristic: the thickness, which is a natural optimization strategy based on reducing dead loads and to minimize construction material. The range of application of this type of structures include aircraft, bridges, ships, storage vessels, industrial buildings, among many others.

There is a considerable amount of research on the stability problem of thin-walled structures by numerical methods such as finite element method (FEM) [1,2], finite strip method (which is basically a special form of FEM) [3], transfer matrix method [4], among others. Bedair [5] presents an extensive review on some of these techniques, with special emphasis on stiffened panels.

The boundary element method (BEM) is a well establish numerical technique in structural mechanics [6]. Manolis et al. [7] developed a direct boundary element formulation dealing with linear elastic stability analysis of Kirchhoff plates. Syngellakis and Elzein [8] extended and refined a plate buckling boundary element formulation to incorporate any combination of loading and support conditions. Nerantzaki and Katsikadelis [9] used the analog equation method (AEM) to solve buckling analysis of plates with variable thickness. According to this method the displacement and its derivatives in the fourth order partial differential equation with variable coefficients are expressed in terms of a fictitious load which is established from the integral equation solution of an adjoint analog equation. Lin et al. [10] developed a more general boundary element formulation for the investigation of the onset of instability of elastic plate, which allowed for wide variety of boundary conditions and arbitrary planar shapes. More recently, Purbolaksono and Aliabadi [11] presented the first BEM plate buckling formulation using a shear deformation theory to solve problems with different loading and boundary conditions; Baiz and Aliabadi presented the first formulations for shallow shell buckling using a boundary-domain element formulation [12] and only boundary element formulation [13], in both cases using a shear deformation theory. Other recent contributions on plate and shell stability by BEM can be found in [14,15].

There are very few studies on plate assemblies using BEM, and to the authors best knowledge no study on shell assemblies have been presented. The first work on plate assemblies with BEM was presented by Tanaka and Miyazaki [16], using a direct formulation for elastic plate structures based on Kirchhoff plate theory and plane stress elasticity. Dirgantara and Aliabadi [17], presented an application of BEM for the analysis of plate structures subjected to different boundary conditions using a shear deformable plate formulation. More recent works on plate assemblies by BEM have been presented by Wen et al. [18] for crack growth analysis in multi-layered airframe structures and by Di Pisa [19] for large deflection, fracture mechanics, debonding, rivets and repairs applications. In the case of linear buckling for assembled plate structures, the only available work has been presented by Tanaka and Miyazaki [20] using classical plate theory (Kirchhoff) with the boundary-volume element method.

The present paper considers the local buckling phenomenon in assembled thin-walled structures (the junctions remain fixed in

* Corresponding author. Tel.: +44 20 7589 5111.
E-mail address: p.m.baiz@imperial.ac.uk (P.M. Baiz).

0955-7997/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2008.07.005

space during buckling), by a boundary element formulation for shear deformation plates and shallow shells where the domain integrals are transferred to the boundary by the use of the dual reciprocity method (DRM). Results are compared with FEM solutions and it is shown that good accuracy can be achieved in the present multi-region BEM formulation, with relatively coarse meshes.

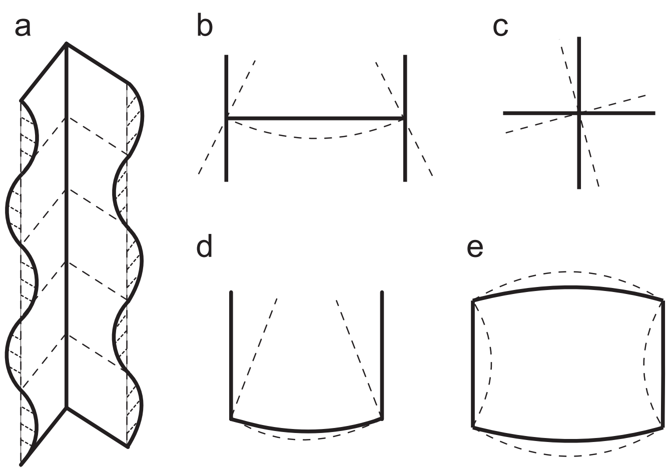

Fig. 1. Typical local distortion of common thin-walled cross sections.

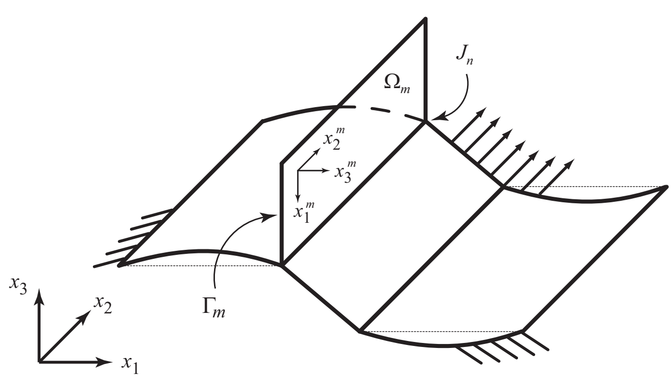

Fig. 2. Assembled shallow shells and plates (thin-walled structures).

2. Buckling of shear deformable thin-walled structures

In thin-walled members, when the internal membrane state created by the external loading is compressive, it is most likely that the structure will fail due to stability problems. Instability in plate and shell assemblies can be studied from different perspectives depending on global, local or interactive buckling affects the behaviour of the structure. The main difference between global and local buckling is in the assumption that in local instability the global axis of the structure remains straight, see Fig. 1. Compressed members usually experience first local buckling effects and depending on the length of the structure (direction in which the structure is compressed) can be combined with global modes. An eigenvalue buckling analysis provides information about the critical load and the most likely modes of collapse. It is for this reason that an eigenvalue analysis is usually carried out as the first step in the stability analysis. The eigenmodes obtained from this analysis can be then used in the investigation of imperfection sensitivity of the structure.

2.1. Shell assembly formulation (multi-region)

Let us consider M assembled cylindrical shallow shells or plates joined at J_n as shown in Fig. 2. The global coordinate system is given by $-x_1-x_2-x_3$, and the local coordinate systems for each region by $-x_1^m-x_2^m-x_3^m$ $(m=1,M)$.

In the simple case of two shallow shells with the same axis orientation at the junction line (see Fig. 3), the continuity and equilibrium equations along the joint can be written as follows:

$$u_\alpha^m = u_\alpha^{m+1}$$
$$w_i^m = w_i^{m+1} \tag{1}$$

for displacements and rotations, and

$$\sum_{m=1}^{M} t_\alpha^m = 0$$
$$\sum_{m=1}^{M} p_j^m = 0 \tag{2}$$

for tractions and moments, respectively.

As shown in Fig. 3, w_α represent rotations of the middle surface, w_3 denotes the out-of-plane displacement, and u_α represent in-plane displacements. The generalized tractions are denoted as p_α due to the stress couples, p_3 due to shear stress resultant and t_α due to membrane stress resultants.

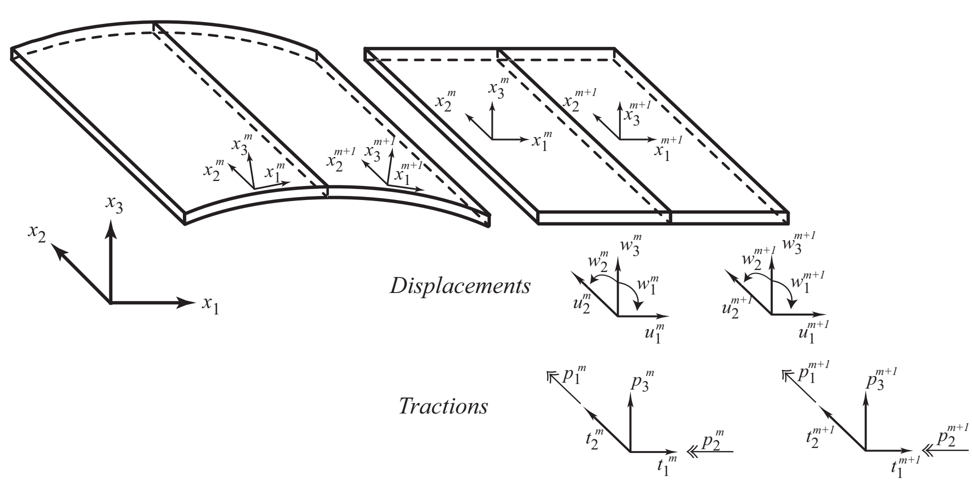

Fig. 3. Simple assemblies of shallow shells or plates.