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A spectral boundary element algorithm for interfacial dynamics in two-dimensional Stokes flow based on Hermitian interfacial smoothing

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Abstract

A two-dimensional spectral boundary element algorithm for interfacial dynamics in Stokes flow is presented. The main attraction of this approach is that it exploits all the benefits of the spectral methods with the versatility of the finite element method. In addition, it is not affected by the disadvantage of the spectral methods used in volume discretization to create denser systems. To achieve continuity of the interfacial geometry and its derivatives at the edges of the spectral elements during the droplet deformation, a suitable interfacial smoothing is developed based on Hermitian-like interpolations. An adaptive mesh reconstructing procedure based on relevant lengths of the spectral elements is also described.

Keywords: Boundary element method; Spectral method; Stokes flow; Interfacial dynamics

1. Introduction

The dynamics of droplets and bubbles in infinite media or in restricted geometries under viscous flows and/or gravity is a problem of great technological and fundamental interest since it is encountered in a broad range of industrial, natural and physiological processes. Industrial applications include enhanced oil recovery, coating operations, vapor condensation, waste treatment, advanced materials processing and microfluidic devices. Pharmaceutical applications include emulsions which serve as a vehicle for the transport of the medical agent to the skin. One further application constitutes the blood flow in microvessels.

Since the pioneering work of Youngren and Acrivos [1] nearly 30 years ago, interfacial dynamics in Stokes flow via the solution of boundary integral equations has developed considerably. The main benefits of this approach are the reduction of the problem dimensionality by one and the great parallel scalability. A lot of research has been done to determine and understand the deformation of droplets and bubbles in external flows, both in infinite media as well as in constrained geometries [2–4]. Considerable progress has also been made in the study of membrane-like interfaces such as those in artificial capsules and biological cells [2,5]. During the last years the interaction of deformable interfaces, e.g. suspensions of droplets, has received a lot of interest [6]. The coming years are expected to witness a growth in the application of interfacial boundary integral solutions in flows in porous media, microfluidic devices and physiological systems due to the increased interest in small scales.

During the last 30 years, several numerical methodologies have been developed for the solution of the boundary integral equations for interfacial dynamics based mainly on low-order interpolation schemes, e.g. see [2,7–10]. In the present paper we develop a high-order spectral boundary element algorithm for interfacial dynamics in Stokes flow. The main attraction of our algorithm is that it exploits all the benefits of the spectral methods (i.e. exponential convergence and numerical stability with increasing the number of discretization points) while the utilization of surface elements permits the study of the most complicated geometries [11–13]. In addition, it is not affected by the disadvantage of the spectral methods used in volume

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discretization; namely, the requirement to deal with dense systems, because in boundary integral formulations the resulting systems are always dense, independent of the form of the discretization. Our method also exploits all the benefits of the boundary element techniques, i.e. reduction of the problem dimensionality and great parallel scalability.

We emphasize that robust spectral boundary element algorithms have been developed for the study of fixed boundary surfaces [14,15], particulate flows [16], and equilibrium interfaces under steady flows [17,18]. Therefore, with the present algorithm we seek to extend the spectral boundary element formulation to the problem of interfacial dynamic evolution.

After the mathematical formulation in Section 2, in Section 3 we present the spectral interfacial discretization. By applying a time advancing scheme, the resulting algorithm is numerically unstable due to geometric discontinuities at the edges of the spectral elements. To avoid the growth of these numerical discrepancies, in Section 4 we develop a suitable interfacial smoothing based on Hermitian-like interpolations which preserves the continuity of the interfacial geometry and its derivatives at the edges of the spectral elements during the droplet deformation. In Section 5 we present the convergence in the numerical accuracy as the number of the employed spectral points increases for the interfacial curvature and for the dynamic evolution of the interfacial shape. An adaptive mesh reconstructing procedure based on relevant lengths of the spectral elements is also described.

In the current paper we present our algorithm for two-dimensional flows. This algorithm can be employed for the study of physical problems which are by nature two-dimensional, i.e. deformation of films and (long) cylindrical fluid volumes [19,20]. It can also be employed to investigate extensively the influence of the length and width of associated three-dimensional problems (even though it cannot investigate the influence of the problem depth). The methodologies presented here can be extended in three-dimensional flows in a straightforward, though non-trivial, manner.

2. Mathematical formulation

We consider a two-dimensional droplet suspended in an infinite fluid as illustrated in Fig. 1. The droplet size is specified by its volume V or equivalently by the radius a of a circular droplet of volume $\pi a^2 = V$. The droplet (fluid 1) has density ρ_1 and viscosity $\lambda\mu$, while the surrounding fluid has density ρ_2 and viscosity μ. The gravitational acceleration is g while the surface tension γ is assumed constant. The undisturbed flow exterior to the droplet is $\boldsymbol{u}^\infty$, e.g. an extensional flow $\boldsymbol{u}^\infty = G(x, -y)$ or a simple shear flow $\boldsymbol{u}^\infty = G(y, 0)$, where G is the shear rate. In this study, the characteristic length a is used as the length scale while the time is scaled with the flow time scale G^{-1}.

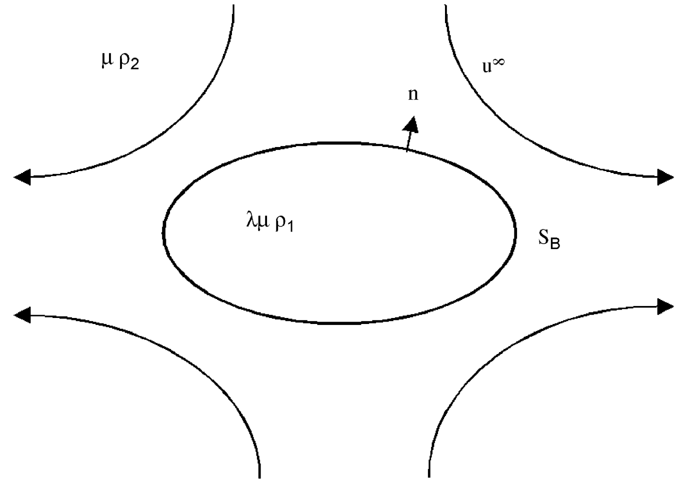

Fig. 1. Fluid droplet suspended in a surrounding fluid.

The capillary number Ca and Bond number B_d are defined by

$$Ca = \frac{\mu G a}{\gamma}, \quad B_d = \frac{(\rho_1 - \rho_2) g a^2}{\gamma}. \tag{1}$$

These dimensionless parameters represent the ratio of viscous flow forces and gravitational forces to interfacial forces, respectively.

The governing equations in fluid 2 are the Stokes equations together with continuity

$$\nabla \cdot \boldsymbol{\sigma} = -\nabla p + \mu \nabla^2 \boldsymbol{u} = 0, \tag{2}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{3}$$

while in the droplet, the same equations apply with the viscosity replaced by $\lambda\mu$.

At the interface, the boundary conditions on the velocity $\boldsymbol{u}$ and surface stress $\boldsymbol{f}$ are

$$\boldsymbol{u}_1 = \boldsymbol{u}_2, \tag{4}$$

$$\Delta \boldsymbol{f} = \boldsymbol{f}_2 - \boldsymbol{f}_1 = \gamma(\nabla \cdot \boldsymbol{n})\boldsymbol{n} + (\rho_2 - \rho_1)(\boldsymbol{g} \cdot \boldsymbol{x})\boldsymbol{n}. \tag{5}$$

Here, the subscripts designate quantities evaluated in fluids 1 and 2, respectively. The surface stress is defined as $\boldsymbol{f} = \boldsymbol{\sigma} \cdot \boldsymbol{n}$ where $\boldsymbol{n}$ is the unit normal which we choose to point into fluid 2. The pressure as defined in $\boldsymbol{\sigma}$ is the dynamic pressure; hence the gravity force is absent from Eq. (2) and appears in the interfacial stress boundary condition, Eq. (5).

The velocity at a point $\boldsymbol{x}_0$ on the droplet surface S_B may be described by the boundary integral equation

$$(1+\lambda)\boldsymbol{u}(\boldsymbol{x}_0) - 2\boldsymbol{u}^\infty(\boldsymbol{x}_0) = -\frac{1}{2\pi\mu}\int_{S_B} [\boldsymbol{S} \cdot \Delta \boldsymbol{f} - (1-\lambda)\mu \boldsymbol{T} \cdot \boldsymbol{u} \cdot \boldsymbol{n}]\, \mathrm{d}S, \tag{6}$$

where S_{ij} is the fundamental solution for the two-dimensional Stokes equations and T_{ijk} the associated stress