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3D low frequency electromagnetic modelling of the human eye with boundary elements: Application to conductive keratoplasty

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Abstract

Conductive keratoplasty (CK) is a non-ablative surgical technique for the treatment of mild to moderate hyperopia (far-sightedness). In a CK session a thin electrode penetrates the cornea and delivers pulsed radio-frequency energy at 350 kHz to the surrounding tissue. The electromagnetic (EM) energy is dissipated into heat in the tissue surrounding the tip yielding thermally localised shrinkage and tightening of the collagen lamellae. When applied in a controlled way, this relatively new technique allows eye surgeons to correct the shape of the cornea and to treat common eye types of disease such as far-sightedness or astigmatism. The purpose of this work is to present a three-dimensional boundary element model of the human eye and its solution for the current density distribution in the different tissues when exposed to CK treatment. This is in order to examine in detail the induced currents that appear in the rest of the eye while delivering the EM signal, and to understand how the human eye behaves as a bulk imperfect conductor when a low frequency high voltage signal is locally applied. The Boundary element method is implemented with a multi-domain staggered approach in which each boundary element contains both continuous and discontinuous nodes. The former allocate electric potential only, while the latter allocate normal electric field only. In this way, continuity of the potential is ensured between adjacent elements and the complication of matching normal electric fields at corner points generated by elements of different orientations is avoided. A quantitative estimation of the EM energy absorbed in each tissue is presented, as well as the initial impedance of the electrodes.

Keywords: Human eye; Boundary element method; Conductive keratoplasty; Electric field

1. Introduction

The human eye is a complex organ capable of detecting light. In a nutshell, the front of the eye can collect, focus and regulate the incident beam, essentially by means of the cornea, pupil, lens, anterior chamber and iris, whereas the back of the eye including retina, macula and optical nerve, can capture the incident light and react by producing a series of electrical impulses.

There is a great deal of interest in understanding the electrical properties of the human eye in the low frequency (LF) range, and how it behaves as a complex imperfect conductive body. The involved studies are oriented to consider the eye as either a passive element exposed to an external field, or as an active one which produces a complicated pattern of electrical signals in response to light. The former is usually boarded in order to understand the vulnerability of the different tissues when exposed to potentially harmful levels of electric fields at different frequencies, orders of magnitude and exposure times [1,2]. While the latter is usually studied in order to understand better how vision works, to develop machines and technologies capable of simulating the electric response of the photosensitive tissues, and to be able to obtain a more accurate interpretation of the patterns of signals coming from electro-oculograms or similar techniques.

On the other hand, the development of safe and effective thermal techniques to alter the topography of the cornea in the eye in order to correct vision defects in people has been challenging ophthalmologists for more than hundred years. It is well known that localised high temperatures can induce shrinkage in the corneal collagen. In addition, an oscillating electric field can be absorbed by biological

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tissues and used as a mean to produce heat in a local, controlled way. Hence, an important application which has increasingly brought attention is the harness of electric fields in order to thermally reshape the cornea in a way to correct defects of vision arising from geometrical defects. A recently successful technique based in this principle is called conductive keratoplasty (CK).

For all the above-mentioned applications, the accurate prediction of the electric field and current densities in the different tissues of the human eye are of paramount relevance. The calculation of LF electric fields in the human eye requires solving the macroscopic Maxwell equations [3] in an assembly of biological tissues with different conductivities and permittivities. For this purpose, numerical modelling is one of the most useful approaches.

In time harmonic electromagnetic (EM) fields, Maxwell equations can be decoupled when the characteristic size of the model (2–3 cm for the human eye) is much smaller than the characteristic vacuum wavelength ($\lambda_0 \sim 860$ m at frequency $f = 350$ kHz) and the displacement currents can be neglected in comparison to the resistive ones. Therefore, the high voltage LF approach is applicable and the numerical problem summarises into solving the non-homogeneous Laplace equation. On the other hand, the boundary element method (BEM) [4,5] is a well-established numerical technique capable of delivering accurate solutions even in complex geometries. The beauty of the formulation is that it takes into account the fundamental solution of the leading differential operator, and that the discretisation can be done only in the boundary of the problem. The BEM has been recently successfully applied for solving problems of biological tissues, other than the human eye, exposed to low and high frequency EM fields [6–9].

The objective of this paper is to present a multi-domain (MD) staggered BEM (S-BEM) approach for computing accurate solutions of LF electric fields in a three-dimensional model of the human eye. This paper pays particular attention to the case of exposure to CK treatment, although the same model and formulation can be developed for solving other problems of exposure.

2. LF EM modelling

When considering biological tissue exposure to high voltage and low intensities systems the most influential field is the electric one. Assuming both conductivity σ and permittivity ε to be constant within a finite region of interest (sub-domain) it is found from Maxwell's equations [3] that the electric scalar potential φ obeys the following non-homogeneous Laplace equation:

$$\nabla \cdot [(\sigma + \mathrm{i}\omega\varepsilon)\nabla\varphi] = 0, \tag{1}$$

where $e^{i\omega t}$-time dependency convention has been used, being $\omega = 2\pi f$ the angular frequency of the incident field, ε the permittivity, and $i^2 := -1$.

At LF, biological tissues behave as good conductors with conductivity values of the order of 0.5 S/m, and electric permittivity 10^{-10} F/m, i.e. $\varepsilon_r \sim 100$; and the air represents a nearly perfect dielectric.

2.1. Interface matching conditions

Both σ and ε are considered constant scalars within each sub-domain $\Omega_s \in \Omega$ of the model (Ω). However, a more accurate description of the human eye should take into account the anisotropy associated mainly to the cornea, because of its laminar structure with preferential orientation.

Fig. 1 shows the interface between two regions (1 and 2) of different material properties. The unit vector $\hat{\mathbf{n}}$ is the normal of the surface dividing the two media.

The charge conservation assumption leads to the following continuity condition through the interface, where the superscripts 1 and 2 indicate the two media

$$\left[(\sigma + \mathrm{i}\omega\varepsilon)\frac{\partial\varphi}{\partial n}\right]_{(1)} = \left[(\sigma + \mathrm{i}\omega\varepsilon)\frac{\partial\varphi}{\partial n}\right]_{(2)}. \tag{2}$$

In general, φ is regarded as a complex potential $\varphi = \varphi_R + \mathrm{i}\varphi_I$. Then (2) can be split into two equations. At LF, conducting properties are dominant, i.e. $\sigma \gg \omega\varepsilon$ for the different biological tissues. Henceforth, Maxwell equations and particularly Eq. (2) can be decoupled. In the case of the interface between air and biological tissue, the former has negligible conductivity in comparison with the latter. At the same time the permittivity of almost any biological tissue is few orders of magnitude greater than ε_0 [10,11], therefore an appropriate matching condition between air and biological tissue can be written as

$$\left[\sigma\frac{\partial\varphi_R}{\partial n}\right]_{(BIO)} = 0,$$
$$\left[\sigma\frac{\partial\varphi_I}{\partial n}\right]_{(BIO)} = \left[\omega\varepsilon\frac{\partial\varphi_R}{\partial n}\right]_{(AIR)}. \tag{3}$$

On the other hand, for interfaces mediating two regions of biological tissue ($BIO1$) and ($BIO2$), the following relations can be derived:

$$\left[\sigma\frac{\partial\varphi_R}{\partial n}\right]_{(BIO1)} = \left[\sigma\frac{\partial\varphi_R}{\partial n}\right]_{(BIO2)}, \tag{4}$$

$$\left[\sigma\frac{\partial\varphi_I}{\partial n}\right]_{(BIO1)} = \left[\sigma\frac{\partial\phi_I}{\partial n}\right]_{(BIO2)}. \tag{5}$$

In other words, the normal current density $\mathbf{j}$ is preserved throughout the interface between regions of different conductivity.

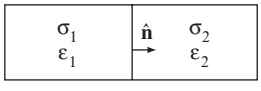

Fig. 1. Interface between two regions of different properties.