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Engineering Analysis with Boundary Elements 30 (2006) 834–846

ENGINEERING ANALYSIS with BOUNDARY ELEMENTS

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Analysis of 3D cracks in anisotropic multi-material domain with weakly singular SGBEM

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Received 3 January 2006; accepted 30 May 2006

Abstract

A weakly singular, symmetric Galerkin boundary element method (SGBEM) is established for analysis of cracks in a three-dimensional, linearly elastic domain consisting of subdomains which are made from different materials. The integral equations governing each subdomain are obtained by employing a pair of weakly singular, weak-form displacement and traction integral equations. A system of governing equations for the entire domain is subsequently obtained in a symmetric form by properly combining the integral equations for each subdomain along with the use of the continuity of displacements and tractions on the subdomain interface. The technique developed possesses several important features: (1) it is applicable for modeling cracks with arbitrary geometry and under general loading; (2) the governing integral equations contain only weakly singular kernels (of $\mathcal{O}(1/r)$) such that their validity requires only the continuity of the displacement boundary data; (3) the formulation is symmetric and gives rise to a system of linear equations with a symmetric coefficient matrix; and (4) the formulation allows the treatment of a multi-material domain where a material constituting each subdomain can be generally anisotropic. In the numerical implementation, standard C^o elements are employed everywhere except along the boundary of the crack surface where special crack-tip elements are utilized to model the asymptotic behavior in the vicinity of the crack front. The use of this crack-tip element allows, additionally, the mixed-mode stress intensity factors to be determined directly and accurately in terms of extra degrees of freedom introduced at nodes along the crack front. Several example problems are presented to demonstrate the accuracy and capability of the technique; numerical results indicate that highly accurate stress intensity factors can be obtained with use of relatively coarse meshes.

Keywords: Boundary elements; Integral equations; weakly singular; Cracks; Multi-material; Anisotropy; Stress intensity factors

1. Introduction

Within the context of linear elastic fracture analysis, the primary quantities of interest are the stress intensity factors which provide a measure of the asymptotic elastic fields in the neighborhood of the crack front. These quantities can be obtained analytically for only problems with relatively simple geometry and loading. In three-dimensional applications involving complex cracks and component geometries, a comprehensive linear fracture analysis requires a computational procedure that can resolve all three stress intensity factors as functions of position along the crack front. Numerical techniques based on boundary integral equations are efficient and attractive for this purpose since only the boundary of the domain and the crack surface require discretization provided that the body is free of body force [1]. Extensive literature review on boundary element methods for three-dimensional, linear fracture analysis can be found in [2,3] for isotropic media and in [4,5] for anisotropic media.

Recently, the symmetric Galerkin boundary element method (SGBEM), based on a pair of weakly singular, weak-form displacement and traction integral equations, has been successfully developed to model cracks in three-dimensional isotropic, linear elastic media [6–8]. In the particular work of Li et al. [7], they utilized the weak-form displacement and traction integral equations developed by Li and Mear [3] to implement the weakly singular SGBEM to model cracks in isotropic media. The important feature

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doi:10.1016/j.enganabound.2006.05.005

of their formulation is that the integral equations involve only weakly singular kernels of $\mathcal{O}(1/r)$ and, as a consequence, validity of integrals requires only the continuity of displacement boundary data. Note that this formulation is equivalent to that developed by Bonnet [9] for an elastic body which is free of cracks. In addition, the weakly singular, weak-form traction integral equation (obtained by Li and Mear [3]) reduces to that by Gu and Yew [10] for isolated planar cracks under pure mode-I loading and to that by Xu and Ortiz [8] for arbitrary cracks under general mixed-mode loading. It can be remarked however that all these formulations are still restricted to a homogeneous body which is made from an isotropic material.

For cracks in generally anisotropic media, the generalization of the weakly singular formulation to treat material anisotropy is nontrivial owing to the complexity of the fundamental solution, and less number of works has been found in the literature. Becache et al. [11] developed a weakly singular traction integral equation for three-dimensional anisotropic elastodynamic problems. In their formulation, the final form of the kernel was expressed in the frequency and Fourier transform domains, and the specialization of these results to static the case appears to be nontrivial. Xu [12] utilized the representation of the relative crack-face displacement in terms of a continuous distribution of dislocation loops to establish a variational traction integral equation. While his formulation is only weakly singular and applicable to general anisotropy, it is restricted to cracks in unbounded domains. Rungamornrat and Mear [5] developed a systematic technique to derive a pair of weakly singular, weak-form displacement and traction integral equations for cracks in three-dimensional anisotropic, linear elastic domains. The key step in their development is the use of special representations for the stress fundamental solution and the strongly singular kernel to allow an integration by parts to be performed via Stokes' theorem. The kernels appearing in their final integral equations are only weakly singular of $\mathcal{O}(1/r)$ and are in a form conducive to the numerical treatment. Rungamornrat and Mear [4,13] developed the SGBEM based on this pair of weakly singular integral equations to model arbitrary cracks in both anisotropic unbounded and finite domains. The method has proven to be efficient and yields highly accurate mixed-mode stress intensity factors with use of relatively coarse meshes; however, the formulation is applicable only to treat cracks in a single material domain.

Certain boundary integral equation methods have been successfully developed, along with the use of a domain decomposition strategy, to solve a boundary value problem associated with a body which consists of several different subdomains. Some of these investigations include that by Gray and Paulino [14] in establishing the SGBEM to treat two-dimensional interface and multi-zone problems and that by Frangi and Maier [15] in developing the SGBEM for linear and nonlinear analysis of fractures in a three-dimensional, zonewise homogeneous medium. Note, however, that both of these formulations are restricted to isotropic media. Other types of the domain decomposition which are attractive for linear fracture analysis are couplings between the SGBEM and the standard finite element method (FEM); the former technique is utilized to model the local region containing cracks while the latter is used to treat the remaining of the domain. Details of these coupling strategies can be found in the work of Frangi and Novati [16] for cracks in isotropic media and in that of Rungamornrat [4] and Rungamornrat and Mear [17] for cracks in generally anisotropic media.

In this paper, the weakly singular SGBEM developed by Rungamornrat and Mear [13] is extended to model cracks in a three-dimensional elastic medium which comprises several homogeneous subdomains; the material constituting each subdomain can be either isotropic or generally anisotropic. This strategy provides a computational tool which possesses a capability for performing linear fracture analysis of a wide class of problems, e.g. structural components, anisotropic rocks, and composite structures which may constitute of several different parts. In the following sections, we first summarize a pair of weakly singular, weak-form displacement and traction integral equations and then establish a symmetric system of integral equations governing a multi-region domain. Next, a brief discussion of the numerical implementation, e.g. crack-tip elements, the numerical integration technique, algorithm for evaluating the kernels for anisotropic materials, and determination of stress intensity factors, is included. Finally, several example problems are presented to demonstrate the accuracy and capability of the technique.

2. weakly singular, weak-form integral equations

Consider a homogeneous, anisotropic linearly elastic body containing cracks as shown schematically in Fig. 1. The domain is enclosed by the total boundary S which

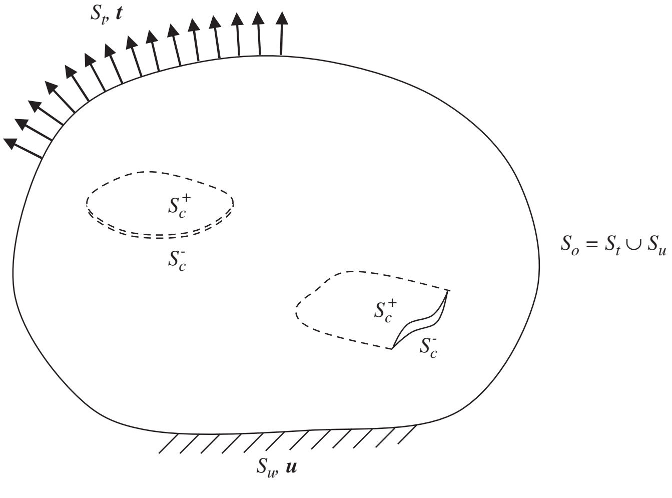

Fig. 1. Schematic of an elastic body containing cracks.