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On the use of piecewise linear wavelets for fast construction of sparsified moment matrices in solving the thin-wire EFIE

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Abstract

Multiresolution wavelet expansion technique has been successfully used in the method of moments (MoM), and sparse matrix equations have been attained. Solving boundary integral equations arising in electromagnetic (EM) problems by the wavelet-based moment method (WMM) involves a time-consuming double numerical integration for each entry of the resultant matrix which in turn can outweigh the advantages of achieving a sparse matrix. The paper presents an alternative computational model to speed up the WMM by excluding double numerical integrations in the evaluation of matrix elements. In this regard, pieces of linear wavelet bases are replaced by proper sinusoidal functions for which closed-form analytical expressions are available. In addition, by introducing approximate closed-form expressions for radiating EM fields of wavelet current elements, the thresholding procedure is modified so that one can compute only the matrix elements of interest. To demonstrate the effectiveness of the proposed method, the thin-wire electric field integral equation (EFIE) is numerically solved by non-orthogonal linear spline wavelet bases.

Keywords: Method of moments; Wavelet expansion; Electric field integral equations; Sinusoidal monopoles; Sparse matrices

1. Introduction

The numerical analysis of electromagnetic (EM) problems is often formulated as differential equations or boundary integral equations or the combination of them as integro-differential equations. To numerically solve the mentioned operator equations, one can initially expand the unknown response by a set of basis functions. Application of appropriate boundary conditions, then, reduces the problem to a matrix equation. In the differential equation approach, the problem under study is solved by using finite difference or finite element methods. Although these methods yield sparse matrix equations, a mesh volume much larger than the interested region is needed to apply an absorbing boundary condition [1]. In the integral equation approach, the computation domain, however, is exactly limited to the boundaries of the interested region. The differential equation formulation can be converted into an integral equation formulation by using Green's function technique [2]. In the integral equation approach, the problem under study is solved by using the boundary element method (BEM), also known as the method of moment (MoM) [3], in which a densely populated matrix equation usually results due to the integral operator.

Recently, the use of wavelet basis functions [4,5] in the MoM known as the wavelet-based moment method (WMM) has attracted much attention [6–28]. The use of wavelets as basis functions in the MoM weakens the mutual interaction of non-overlapping bases and diminishes strong correlation among the expansion coefficients. This is due to numerous useful intrinsic features of wavelets, including natural support for multiresolution analysis (MRA), localization in both the spatial and spectral domains, and the vanishing moment property [4,5]. As a result, the WMM yields to a sparse matrix equation after thresholding [6–28]. Consequently, employing sparse storage schemes and fast sparse-based iterative solvers in solving such a sparse matrix equation drastically reduces the memory requirement and computation time

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[29]. Hence, the merits of sparse matrices attained using the WMM become more apparent in solving large-scale problems [6–24].

The wavelet bases have been applied to integral equations by direct and indirect approaches. In the indirect approach, the discrete wavelet (packet) transform is applied to the matrix obtained using the conventional MoM [7–12]. In the other one, direct approach, shifted and dilated forms of a scaling function and its corresponding wavelet are employed as expansion and test functions [13–28]. Since the direct approach allows to identify prior location of negligible matrix elements, and thus, to avoid the evaluation of the full matrix [13–16], it might be more attractive in practice.

Several wavelet bases have been utilized for solving integral equations. The simplest wavelets, so-called Flatlets, are defined by a set of piecewise constant functions [6]. The well-known Haar wavelet is the simplest example for Flatlet family. Although Flatlets have the lowest vanishing moment, as a result of their narrow support and simple closed-form expression, they offer remarkable improvements [13,26]. Nonetheless, they do not produce a continuous solution. Actually, the continuous, real-valued and compactly supported orthonormal wavelet does not exist [4]. To represent the continuous final solution smoothly, the orthonormal linear spline Battle–Lemarie (Franklin) [25,26] and cubic spline Battle–Lemarie wavelets [13,14] have been used. In many practical EM problems, however, the solution domain is confined to a bounded interval, whereas these wavelets are originally defined on the entire real line extending from $-\infty$ to $+\infty$ [6]. Since amplitudes of the Battle–Lemarie scaling function and wavelet decay fast, the constraint raised by their infinite supports can be relieved by truncation so that one can readily use their periodic version [14]. The periodic wavelets, however, are not able to describe unknown functions with non-equal values at the two endpoints of an interval. Thus, the intervallic wavelets, Daubechies [15] and Coifman (Coiflet) [16] have been utilized to release the endpoint restrictions imposed on the periodic wavelets.

The higher the order of vanishing moments is, the sparser matrices may be attained. The order of vanishing moments usually increases with smoothness. The higher the smoothness of wavelets is, however, the longer their supports in space are. The broader the extent of the wavelet is, the more computational cost should be paid for construction of the moment matrix. This constraint may be removed by the biorthogonal wavelet bases with a number of different orders of vanishing moments for primal wavelets and different smoothness for the dual wavelets [17]. Non-existence of closed-form expression causing the computation and storage of the interior and numerous boundary wavelets and scaling functions (i.e., left and right edges basis functions) degrades the efficiencies of above-mentioned wavelets.

Semi-orthogonal wavelets for which bases in the same subspace do not have orthogonality, are constructed by cardinal *B*-spline functions especially for bounded intervals [5]. Moreover, they have all desirable properties including compact support, closed-form, symmetry, total positivity and small optimal value of space-frequency window product together [18]. Among various types of semi-orthogonal wavelets, the linear one which is continuous up to first-order derivative, and thus, has second-order vanishing moments, is widely used in computational EMs [18–27]. This is due to the fact that using smoother wavelets with higher order of vanishing moments although slightly increases the sparsity of the resultant matrix; it causes larger support in space for wavelets which in turn increases the computational burden in construction of the matrix. As an example, Coifman wavelet with zero moment for its scaling function, covers at least 11 adjacent length scales [16]. In this paper, the compactly supported non-orthogonal linear spline (NLS) wavelet with two vanishing moments [28] is chosen, since its scaling function and wavelet not only possess all eminent features of the semi-orthogonal one, but they also have the minimal support and occupy only two length scales in each resolution level.

Owing to the use of hybrid basis functions in various resolution levels, high computational cost of matrix fill process become more apparent in the WMM than that in the conventional MoM. In fact, even for the piecewise linear wavelets, consisting of linear functions, the time-consuming double numerical integrations in evaluation of dominant matrix entries still outweigh the advantages of achieving sparse matrix in the direct approach. Alternatively, piecewise sinusoidal basis functions are commonly used in the conventional MoM to take the efficiency of closed-form integration [30–37].

The objective of this paper is to speed up the construction of sparse matrices in the WMM. In this regard, in order to directly determine the majority of dominant matrix elements without performing two-fold Gaussian quadrature procedures, each piece of NLS wavelet bases is individually replaced by an appropriate sinusoidal function for which closed-form analytical expressions are available. In addition, the paper presents closed-form expressions for radiated EM fields of wavelet current elements by which one can not only evaluate the left dominant matrix entries with performing just single numerical integrations, but also compute the left single numerical integrations only for the entries of interest even without any knowledge about the sparse structure of the resulting matrix. Note that, in most previous applications of wavelets, the coefficient matrix is calculated as a whole before being thinned by a thresholding procedure.

The remainder of the paper is organized as follows. The next section presents a summery of the thin-wire electric field integral equation (EFIE) formulation. In Section 3, first, the multiresolution wavelet expansion method is reviewed briefly, followed by the general formulation of the WMM. An appropriate piecewise linear wavelet basis choice is, then, selected to better illustrate the new computational model. In Section 4, the sinusoidal