ELSEVIER

Engineering Analysis with Boundary Elements 30 (2006) 687–696

ENGINEERING ANALYSIS with BOUNDARY ELEMENTS

www.elsevier.com/locate/enganabound

Computation of 3D MEMS electrostatics using a nearly exact BEM solver

S. Muhkopadhyay*, N. Majumdar

Saha Institute of Nuclear Physics, 1/AF, Sector 1, Bidhannagar, Kolkata 700064, West Bengal, India

Received 11 August 2005; accepted 10 March 2006
Available online 15 May 2006

Abstract

The electrostatic properties of thin plate shaped structures relevant to the micro-electro-mechanical systems (MEMS) have been computed using a nearly exact boundary element method (BEM) solver. The solver uses closed form expressions for three-dimensional potential and force fields due to uniform sources/sinks distributed on finite flat surfaces. The expressions have been validated and, being analytical, have been found to be applicable throughout the physical domain. The solver has been applied to compute accurately and efficiently the charge densities on thin plate shaped conductors as used in MEMS components. We have presented results for the model problem of parallel plate capacitors and compared them with results obtained from several other BEM based solvers.

Keywords: Boundary element method; Green function; Electrostatics; MEMS; Thin plates

1. Introduction

The boundary element method (BEM) has been successfully applied to various branches of science and technology including gravitation, fluid mechanics, acoustics, structural mechanics, electronics and micro-electro-mechanical systems (MEMS). The method can be viewed as the numerical implementation of boundary integral equations (BIEs) based on the Green's formula. In order to carry out the implementation, only the boundaries need to be segmented. The resulting boundary elements are endowed with distribution of singularities such as sources, doublets, dipoles and vortices. The strength of these singularities are obtained by satisfying the boundary conditions (Dirichlet, Neumann or Robin). Thus, in comparison to finite element method (FEM) and finite difference method (FDM), numerical discretization for BEM is carried out at a reduced spatial dimension and the resulting linear system of equations are smaller. Moreover, due to the nature of Green's formula, the method works accurately for problems with unbounded domains without invoking truncation and other approximations necessary for FEM and FDM. There are several other advantages of using the BEM, e.g., mesh adjustment for moving boundaries is easier to carry out, no interpolation/extrapolation is required for obtaining properties at an arbitrary point in the domain. There are some disadvantages associated with the method as well. For example, the mathematics related to the formulation of a BEM solver is considerably more complex than that for FDM and FEM solvers. Moreover, the matrices generated through this formulation are fully populated and difficult to both generate and solve [1,2]. The most serious drawbacks, however, are related to the approximations involved in the numerical implementation of the BEM. They are, in general, as follows:

- While computing the influences of the singularities, the singularities are modeled by a sum of known basis functions with constant unknown coefficients. For example, in the constant element approach, the singularities are assumed to be concentrated at the centroid of the element, except for special cases such as self influence. This becomes necessary because closed form expressions for the influences are not, in general, available for surface elements. An approximate and

*Corresponding author. Fax: +91 33 23374637.
E-mail address: supratik.mukhopadhyay@saha.ac.in (S. Muhkopadhyay).

0955-7997/$ - see front matter
doi:10.1016/j.enganabound.2006.03.002

computationally rather expensive way of circumventing this limitation is to use numerical integration over each element or to use linear or higher order basis functions.
- The strengths of the singularities are solved depending upon the boundary conditions, which, in turn, are modeled by the shape functions. For example, in the constant element approach, it is assumed that it is sufficient to satisfy the boundary conditions at the centroids of the elements. In this approach, the position of the singularity and the point where the boundary condition is satisfied for a given element usually matches and is called the collocation point.

It may be noted here that the majority of implementations based on BEM are still the constant collocation approaches. This is because the method is a good optimization between accuracy and computational complexity. Both these approximations, especially the former, have important consequences. It leads to the unhappy situation where the potential and force field very close to the element are found to suffer from gross inaccuracies. So much so that a number of workers have had to devise ways of dealing with special cases through which the errors can be reduced to an acceptable extent [3–6]. However, these special formulations are not valid for the entire physical domain and thus, necessitate the use of several expressions for evaluating potential and field on the charged surfaces and other field points. It also becomes necessary to subdivide the segments on some parts of the boundary to justify certain approximations used to deduce the expressions.

Accuracy of the near-field calculations becomes exceedingly important for the structures used in MEMS which normally have fixed or moving structures with thickness (h) of the order of microns (μm) and lengths (L) of the order of tens or hundreds of microns. These structures are often plates or array of thin beams which, owing to their smallness, can be moved or deflected easily through the application of low voltages and are widely used in microjets, microspeakers, electrostatic actuators etc. Since electrostatic forces play a very major role in maneuvering these devices, a thorough understanding of the electrostatic properties of these structures is of critical importance, especially in the design phase of MEMS. In many cases, the electrostatic analysis of MEMS is carried out using BEM, while the structural analysis is carried out using FEM [7]. In this paper, we concentrate on the computation of the electrostatic properties (e.g., the charge distribution and the capacitance) of thin plates relevant to MEMS using BEM.

It has already been noted [6] that computing the resultant charge due to the two surfaces of a plate under the usual assumption of vanishing thickness of the plate is not an acceptable approach for these structures. This is so because the electrostatic force acting at any point on these surfaces depends on the square of the charge density at that point. The two surfaces of the plate being too near, the standard BEM does not work satisfactorily and several modified BEM have been developed, such as the enhanced BEM and the gradient BIE technique leading to the thin plate BEM [6]. The former is suitable for moderately thick plates, while the latter is suitable for very thin plates, $h/L \leqslant 10^{-3}$.

In this work, we present a BEM solver which overcomes the first approximation mentioned earlier by using closed-form expressions for computing influences that are valid for sources distributed uniformly over a flat surface. As a result, the first (and possibly, the most damaging) approximation for this solver can be relaxed and restated as,

- The singularities distributed on the boundary elements are assumed to be uniform on a particular element. The strength of the singularity may change from element to element.

Thus, our approximation turns to be far less stringent than the necessity to assume the singularity to be concentrated at one single point on an element. This relaxation has profound consequences for the solver and to reflect this fact, we have named the new solver a nearly exact BEM solver. Since the expressions used in this solver are analytic and valid for the complete physical domain, and no approximations regarding the size or shape of the singular surface have been made during their derivation, its application is not limited by the proximity of other singular surfaces or their curvature. In other words, it is possible to apply the solver for solving a very wide range of problems including those containing extremely closely spaced singular surfaces of any shape and size which are known to create major problems for other approaches developed and used so far. Thus, now, the same solver can be easily applied to compute properties for plates of any thickness and applicable for any structure, including those relevant to MEMS, for which electrostatic properties need to be computed. The necessity for special formulations to tackle the near-field domain does not arise at all. The approach, naturally, is not limited to electrostatics only but can be applied to many other fields of science and engineering where similar formulations are obtained.

In Section 2, we have presented a brief discussion on the theory of BEM and the new analytic expressions for the potential and the force field for a flat element having source uniformly distributed on it. In Section 3, we present the numerical implementation of the BEM solver, especially in the context of the MEMS thin plates. The first part of Section 4 contains the validation of the new expressions. Here we also study the effect of using the new expressions vis a vis implementing the usual BEM approximation. In the second part, we present the computation related to plates for a wide range of h/L and d/L where d is the distance between the facing sides of the two plates of a parallel plate capacitor. Here, as in [6], we concentrate on the charge density and the capacitance of the structures,