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Null-field Integral Equations for Stress Field around Circular Holes under Antiplane Shear

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Abstract

In this paper, we derive the null-field integral equation for a medium containing circular cavities with arbitrary radii and positions under uniformly remote shear. To fully capture the circular geometries, separate expressions of fundamental solutions in the polar coordinate and Fourier series for boundary densities are adopted. By moving the null-field point to the boundary, singular integrals are transformed to series sums after introducing the concept of degenerate kernels. The solution is formulated in a manner of a semi-analytical form since error purely attributes to the truncation of Fourier series. The two-hole problems are revisited to demonstrate the validity of our method. The bounded-domain approaches using either displacement or stress approaches are also employed. The proposed formulation has been generalized to multiple cavities in a straightforward way without any difficulty.

Keywords: antiplane deformation; null-field integral equation; degenerate kernel; Fourier series; circular hole; Laplace problem

1. Introduction

Researchers and engineers have paid much attention on the development of boundary integral equations (BIEs) and boundary element method (BEM) since Rizzo [15] proposed a numerical treatment for elastostatics. Most of the efforts have been focused on the singular boundary integral equation for problems with ordinary boundaries. In some situations, the singular boundary integral equation is not sufficient, *e.g.* degenerate boundary, fictitious frequency and spurious eigenvalue. Therefore, the hypersingular equation is required. The role of hypersingularity in computational mechanics has been examined in the review article of Chen and Hong [5]. In the past, several regularizations for singularity and hypersingularity were offered to handle it in direct and indirect ways. Hong and Chen [4] have developed the theory of dual boundary integral equation method (BIEM) and dual BEM with hypersingular kernels. The analytical formula reveals the jump behavior of double layer potentials. How to determine accurately the free terms has received more attentions in the past decade. Two conventional approaches were employed to regularize the singular and hypersingular integrals. First, Guiggiani [9] has derived the free terms for Laplace and Navier equations using bump approach in Fig. 1 (a). Second, Gray and Manne [8] have employed a limiting process to ensure a unique solution from an interior point to boundary in Fig. 1 (b). In the present approach, we employed the degenerate kernel for the two-point fundamental solution in the problems with circular boundaries as shown in Fig. 1 (c).

More recently, Honein *et al.* have solved problems of two arbitrary circular holes or rigid inclusions [10–12] of different shear moduli under uniformly remote shear. They have introduced the Möbius transformations involving the complex potential to analytically investigate the stress field around the hole. The extension to more than two holes may have difficulty in the Honein's formulation. To search a systematic method for multiple circular holes is not trivial. It is found that the tangential shear stress $\sigma_{\theta z}$ at the closest points of the two circular holes tends to infinity as the two cavities approach each other. Mogilevskaya and Crouch [14] have solved the problem of an infinite plane containing arbitrary number of circular inclusions based on the complex singular integral equation. In their analysis procedure, the unknown tractions are approximated by using complex Fourier series. The advantage of their method is that one can tackle a lot of inclusions even inclusions touching one another. However, for calculating an integral over a circular boundary, they did not

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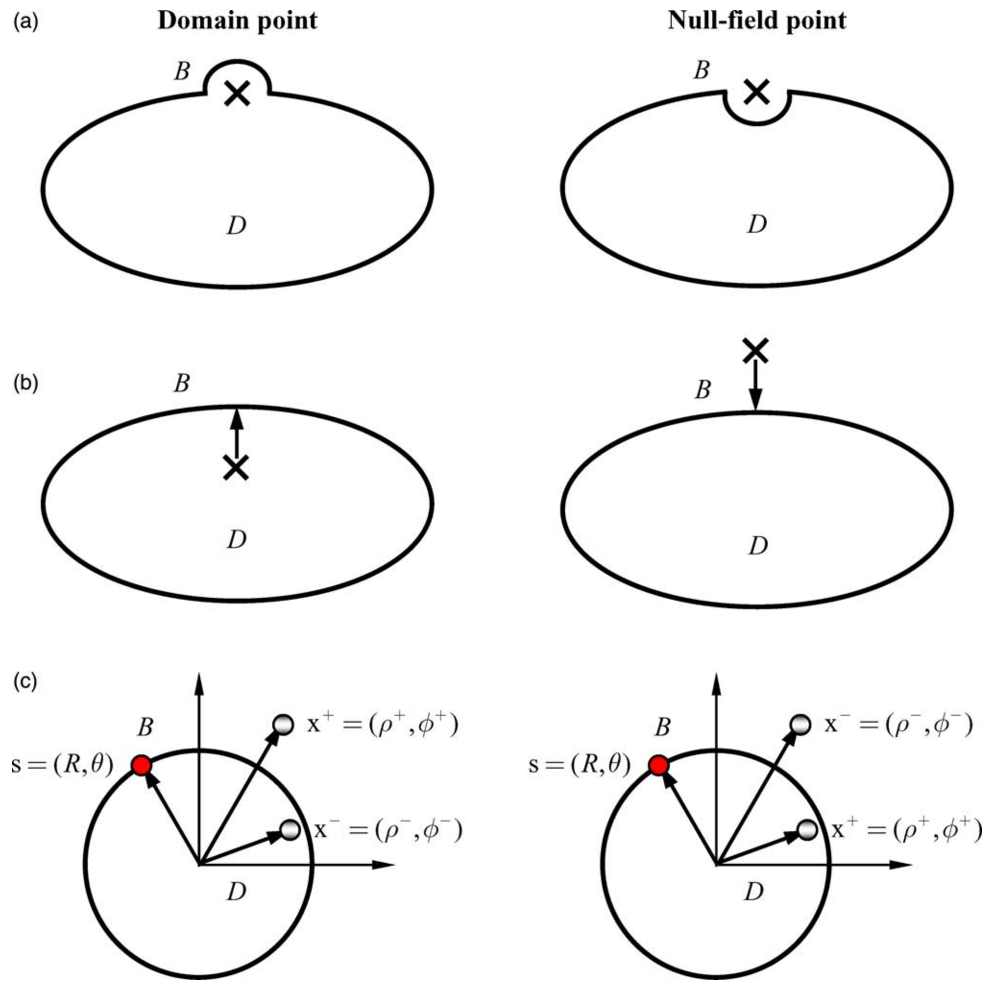

Fig. 1. (a) Bump contour, (b) Limiting process, (c) Present method.

express the fundamental solution using local polar coordinates. By collocating the null-field point on the boundary, the singular integral can be easily determined using series sums in our formulation due to the introduction of degenerate kernels.

Since the analytical solution for more than two cavities may encounter difficulty, several numerical approaches have been employed, *e.g.* complex variable boundary element method (CVBEM) by Chou [7] and Ang and Kang [1]. The CVBEM was primarily introduced by Hromadka and Lai [13] for solving the Laplace problems in an infinite domain. In 1997, Chou extended the work of Hromadka to the multiply connected domain in an infinite plane. The antiplane problem with holes can be formulated in terms of derivatives of displacement. Thus, the stresses around the hole had been obtained directly without resource to numerical differentiation. Recently, Ang and Kang [1] developed a general formulation for solving the second-order elliptic partial differential equation for a multiply-connected region in a different version of CVBEM. The Cauchy integral formulae are offered to solve the exterior boundary value problem [2]. By introducing the CVBEM, Chou [7] and Ang and Kang [1] have revisited the problems with two circular holes whose centers lie on the x axis investigated by Honein *et al.*. Bird and Steele [3] used a Fourier series procedure to solve the antiplane elasticity problems in Honein's paper [11]. To approximate the Honein's infinite problem, an equivalent bounded-domain approach with stress applied on the outer boundary was utilized. A shear stress σ_{rz} on the outer boundary is used in place of a stress σ_{32} at infinity to approach the Honein's results as the radius becomes large.

Since the cavity or inclusion is circular, we may wonder why not using the degenerate kernels in the polar coordinate and Fourier series to best fit the geometry. In this paper, a semi-analytical approach is successfully developed to carry out the solution of the problem under antiplane shear. The mathematical formulation is derived by using degenerate kernels for the fundamental solution and Fourier series expansions for the boundary densities in the null-field integral equation. Then, it is reduced to a linear algebraic system after collocating points on each circular boundary and substituting the boundary conditions. The unknown coefficients in the algebraic system can be determined. Furthermore, arbitrary number of circular holes can be treated by using the present method without any difficulty. A general purpose program for arbitrary number of holes with various radii and different positions was developed.