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Effectiveness of GMRES-DR and OSP-ILUC for wave diffraction analysis of a very large floating structure (VLFS)

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Abstract

This paper presents the performance of iterative solvers and preconditioners for the non-Hermitian dense linear systems arising from the boundary value problem related to the diffraction wave field around a very large floating structure (VLFS). These systems can be solved iteratively using the GMRES with deflated restarting (GMRES-DR), which has Krylov subspaces with approximate eigenvectors as starting vectors. The number of iteration needed by GMRES or GMRES-DR can be significantly reduced using preconditioning techniques. Matrix-vector products are approximated by utilizing the fast multipole method (FMM), which need not directly calculate the dense matrix of the far field interactions. The combination of the operator splitting preconditioner (OSP) and the Crout version of the incomplete LU factorization (ILUC) does not require the dense matrix of the far field interactions. Numerical experiments from a hybrid-type VLFS, which is composed of pontoon-part and semi-submersible part, whose length is 3000 m are presented.

Keywords: Iterative methods; GMRES; Boundary element method; Preconditioning; Operator splitting; Incomplete LU factorization; Fast multipole method; VLFS

1. Introduction

The problem of the interaction between the global structural response of a very large floating structure (VLFS) and the associated diffraction wave field in an unbounded exterior domain is achieved here by matching a boundary element analysis of the exterior diffraction wave field with a finite element analysis of the elastic structure at the fluid-structure interface. We finally arrive at solving a large system of linear equations of the form

$$Ax = b, \tag{1}$$

where the coefficient matrix A is non-Hermitian and dense (see Refs. [1,2]).

The generalized minimal residual method (GMRES) [3] is a well-known iterative method for solving large non-Hermitian linear systems of equations. Since GMRES becomes increasingly expensive and requires more storage as the iteration proceeds, it generally uses restarting, which slows the convergence. The GMRES with deflated restarting (GMRES-DR) [4] is derived from restarting implicitly with an appropriate vector, which is a linear combination of the approximate eigenvectors obtained from the orthogonal projection method [5] onto Krylov subspaces [3–5]. The deflation of eigenvalues can greatly improve the convergence rate of restarted GMRES.

It is well known that the convergence rate of Krylov subspace methods for linear equations depends on the spectrum of A [6]. It is therefore natural to try to transform the original system (1) into one having the same solution but more favorable spectral properties. A preconditioner is a matrix that can be used to accomplish such a transformation. The operator splitting preconditioner (OSP) [7,8] is an effective technique in solving dense linear systems arising from the boundary element method (BEM). OSP splits the coefficient matrix A into the sparse matrix A_{near} of the near field interactions and the dense matrix A_{far} of the far field interactions. Out of A_{near}, the preconditioner M is constructed using the Crout version of the incomplete LU factorization (ILUC) [9]. Matrix-vector products are approximated by utilizing the fast multipole method (FMM) [10,11], which need not directly calculate A_{far}. The OSP-ILUC preconditioner does not require A_{far}. Therefore, it can be expected that the boundary element method using FMM will be further accelerated by OSP-ILUC.

In the first part of this paper, we apply GMRES-DR to the analysis of the boundary value problem related to

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the diffraction wave field around a very large floating structure (VLFS).

In the second part, we apply OSP-ILUC to our example. OSP has a parameter r, which is the minimum distance of far points. We show that OSP becomes ineffective if r is too large. We clarify the range of r where OSP is effective at the same time.

2. Integral equation formulation

Example that is investigated is shown in Fig. 1, which is a hybrid-type VLFS, which is composed of pontoon-part and semi-submersible part, floating in the open sea with constant depth h, whose length is L, whose width is B, and whose draft is d [1]. The coordinate system is defined such that the xy plane locates on the undisturbed free surface and the z-axis points upward. The longcrested harmonic wave with small amplitude is considered. The amplitude of the incident wave is defined by A, the circular frequency by ω, and the angle of incidence by β. $\beta=0$ corresponds to the head wave from positive x direction and $\beta=\pi/2$ to the beam wave from positive y direction.

Assuming the water to be perfect fluid with no viscosity and incompressible, and the fluid motion to be irrotational, then the fluid motion can be represented by a velocity potential Φ. Also, we consider the steady-state harmonic motions of the fluid and the structure, with the circular frequency ω. Then, all of the time-dependent quantities can be represented similarly as follows:

$$\Phi(x,y,z,t) = \mathrm{Re}[\phi(x,y,z)\mathrm{e}^{\mathrm{i}\omega t}], \tag{2}$$

where i is the imaginary unit and t is the time.

Assuming the fluid motion to be small, we can employ the linearized wave theory. Then, our problem is formulated as the boundary value problem represented by a velocity potential ϕ as follows:

$$\nabla^2\phi = 0 \quad \text{in } \Omega \tag{3}$$

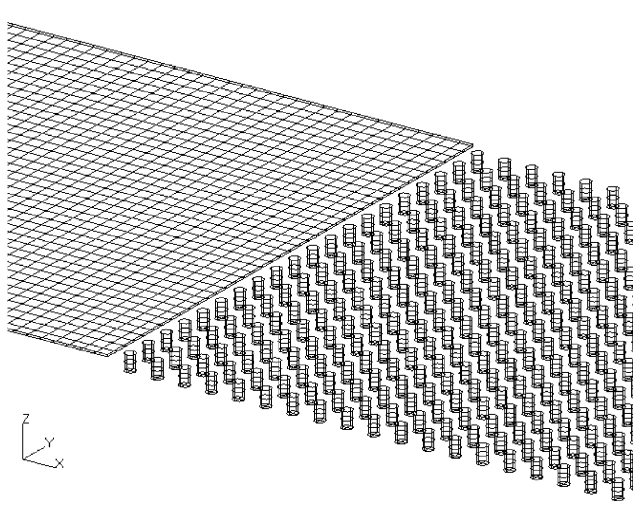

Fig. 1. A hybrid-type VLFS [1].

$$\frac{\partial\phi}{\partial z} = K\phi \quad \text{on } S_{\mathrm{F}} \tag{4}$$

$$\frac{\partial\phi}{\partial z} = 0 \quad \text{on } B_0 \tag{5}$$

$$\frac{\partial\phi}{\partial n} = 0 \quad \text{on } S_{\mathrm{H}} \tag{6}$$

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial(\phi-\phi_I)}{\partial r} - \mathrm{i}k(\phi-\phi_{\mathrm{I}})\right) = 0 \quad \text{on } S_\infty \tag{7}$$

$$\phi_{\mathrm{I}} = \mathrm{i}\frac{gA}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\mathrm{e}^{\mathrm{i}k(x\cos\beta + y\sin\beta)} \tag{8}$$

Here, ϕ_{I} is the incident wave potential. The fluid domain is defined by the symbol Ω, the undisturbed free surface by S_{F}, the flat bottom base surface of $z=-h$ by B_0, the wetted-surface of VLFS by S_{H}, and the boundary at infinity by S_∞. The symbol n represents the normal vector (positive direction is out of fluid domain into the body inner part), and r is a horizontal distance from the origin. The symbol K is the wave number in the infinite depth ($=\omega^2/g$; g is the gravitational acceleration), and k is the wave number. Then, the following dispersion relation is satisfied:

$$k\tanh kh = K \tag{9}$$

Substituting the boundary conditions represented by a velocity potential ϕ into the integral equation, we have the following integral equation [12]:

$$4\pi\phi(x) + \int_{S_{\mathrm{H}}}\left\{\phi(\xi)\frac{\partial G(x,\xi)}{\partial n_\xi} - \phi(x)\frac{\partial G_2(x,\xi)}{\partial n_\xi}\right\}\mathrm{d}\xi = 4\pi\phi_{\mathrm{I}}(x) \tag{10}$$

Here, $G(x,\xi)$ is called the free-surface Green's function and it is a fundamental solution of the Laplacian ∇^2 which satisfies boundary conditions (4), (5) and (7), and $G_2(x,\xi)$ is the auxiliary Green's function which corresponds to a rigid free surface condition [12]. From the integral Eq. (10) we arrive at a system of linear equations, whose solution is the potentials at nodes after discretization of the S_{H} surface [1,11].

3. GMRES with deflated restarting

It is well known that the convergence rate of GMRES for linear equations depends on the distribution of eigenvalues of the coefficient matrix A. Therefore, deflating small eigenvalues can greatly improve the convergence rate. The GMRES with deflated restarting (GMRES-DR) [4] is derived from restarting implicitly with an appropriate vector, which is a linear combination of harmonic Ritz vectors [4].

3.1. Harmonic Ritz pairs

Definition 1. Let A be an $n\times n$ complex matrix and T be an m dimensional subspace of $\mathbf{C}^n$. A pair (θ,y), where θ is a complex number and y is a non-zero vector in T, is called a Ritz pair