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Modeling of the ground plane in electrostatic BEM analysis of MEMS and NEMS

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Abstract

This paper is concerned with calculation of charge distribution on conducting elements in microelectromechanical and nanoelectromechanical systems (MEMS and NEMS). The conductors are beam like in MEMS or nanotubes in NEMS. The ground plane is typically present in such problems. This is usually modeled by constructing image elements with the ground treated as a flat mirror, thereby doubling the size of a problem. An alternative approach is to model the ground directly in a boundary element method (BEM) formulation of the problem. The latter approach is adopted in this paper. The governing BEM equations, and their regularization, is discussed in detail. Numerical results are presented for selected examples and their results are compared with analytical solutions whenever possible.

Keywords: Boundary element methods; Microelectromechanical systems; Nanoelectromechanical systems; Beams; Nanotubes; Modeling of ground

1. Introduction

Microelectromechanical systems (MEMS) have demonstrated important applications in a wide variety of industries including mechanical and aerospace, medicine, communications, information technology etc. Nanoelectromechanical systems (NEMS) are "smaller" MEMS in the sense that they have submicron critical dimensions. Owing primarily to their small size, NEMS can offer very high sensitivities (e.g. force sensitivities at the attonewton level, mass sensitivities at a single molecule or even a single atom level, and charge sensitivities at the level of the charge on a single electron). In addition, they offer mechanical quality factors in the tens of thousands and fundamental frequencies in the microwave range [1,2]. Fabrication of silicon nanotweezers [3] and nanoresonators [4] has been demonstrated recently. Carbon nanotubes (CNTs) have remarkable properties—they are very stiff, have low density, ultra-small cross-sections and can be defect free. They offer fascinating applications possibilities. Natural frequencies of tunable CNTs have been measured recently [5].

Numerical simulation of electrically actuated MEMS devices have been carried out for around a decade or so by using the boundary element method (BEM—see, e.g. [6–10]) to model the exterior electric field and the finite element method (FEM—see, e.g. [11–13]) to model deformation of the structure. The commercial software package MEMCAD [14], for example, uses the commercial FEM software package ABAQUS for mechanical analysis, together with a BEM code FastCap [15] for the electric field analysis. Other examples of such work are [16–19]; as well as [14,20,21] for dynamic analysis of MEMS. A very nice recent example of NEMS simulation is [22]. This paper employs the classical electrostatic model for nano conductors and three different electrostatic models: classical, semiclassical and quantum-mechanical, for semiconductors.

Many applications in MEMS and NEMS require BEM analysis of the electric field exterior to thin conducting objects. In the context of MEMS with very thin beams or plates (see Fig. 1), a convenient way to model such a problem is to assume plates (or beams) with vanishing

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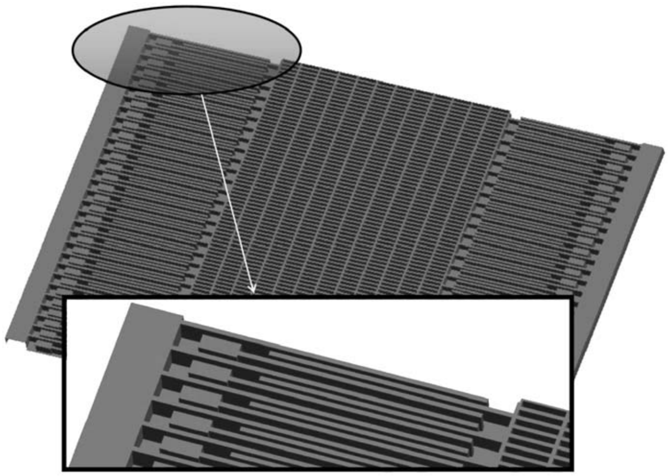

Fig. 1. Parallel plate resonator: geometry and detail of the parallel plate fingers (from [23]).

thickness and solve for the sum of the charges on the upper and lower surfaces of each plate [24] (or beam). The standard boundary integral equation (BIE) with a weakly singular kernel is used in [24] and this approach works well for determining, for example, the capacitance of a parallel plate capacitor. For MEMS calculations, however, one must obtain the charge densities separately on the upper and lower surfaces of a plate (or beam) since the traction at a surface point on a plate (or beam) depends on the square of the charge density at that point. The gradient BIE is employed in [25] to obtain these charge densities separately. The formulation given in [25] is a BEM scheme that is particularly well-suited for MEMS analysis of very thin plates—for $h/L \leqslant 0.001$—in terms of the length L (of a side of a square plate) and its thickness h. A similar approach has also been developed for MEMS with very thin beams [26]. Similar work has also been reported recently by Chuyan et al. [27] in the context of determining fringing fields and levitating forces for 2D beam shaped conductors in MEMS combdrives. A fully coupled BEM/FEM MEMS calculation with very thin plates has just been completed [28]. See, also, [29] for an application of the thin plate idea for modeling damping forces on MEMS with thin plates.

Another interesting problem that has been solved recently deals with charge distribution on thin conducting CNTs [30]. A line model for a nanotube is proposed here. As before for beams and plates, this model overcomes the problem of dealing with nearly singular matrices that occur when the standard BEM is applied to very thin features (objects or gaps). The charge distribution per unit length on the surface of a nanotube is obtained first. The charge density per unit area on the entire nanotube surface is obtained next by a post-processing step. This new approach is very efficient. Numerical results are presented for various examples. Results for selected problems are compared with those from a full 3D BEM calculation [31] as well as with available analytical solutions [32]. Excellent agreement is obtained with available analytical solutions.

In MEMS and NEMS calculations of the type considered here, ground planes are typically present. The typical approach for dealing with this issue is to construct an image plane with the ground plane treated as a flat mirror. This, of course, doubles the size of a given problem. The BEM formulation is modified in the present work so that the ground plane is modeled directly. Problems with conducting beams are considered first. This is followed by problems with thin conducting nanotubes.

An alternative way of modeling the ground plane is to use half-space Green's functions in the BEM formulation. This approach is presented in [30] for nanotube problems. One advantage of the approach discussed in the present paper is that, unlike in other formulations, the charge distribution on the ground is also obtained here. This information can be useful in certain problems.

The present paper is organized as follows BIEs are first presented for an infinite region containing two thin conducting beams. This is followed by BIEs in a semi-infinite region containing one thin conducting beam and the ground. The next section deals with a semi-infinite region containing an arbitrary number of beams, together with the ground plane. Each beam can be of arbitrary length and be oriented in an arbitrary direction, but their mid-planes must all lie in the same plane. Numerical results are presented for some beam problems. Analysis of conducting nanotubes follows using the same format as described above. A concluding remarks section completes the paper.

2. BIEs in infinite region containing two thin conducting beams

Consider the situation shown in Fig. 2 with two parallel beams. Of interest is the solution of the following Dirichlet problem for Laplace's equation:

$$\nabla^2\phi(\mathbf{x}) = 0, \quad \mathbf{x} \in B, \quad \phi(\mathbf{x}) \quad \text{prescribed for } \mathbf{x} \in \partial B, \tag{1}$$

where ϕ is the potential and B is the region *exterior* to the two beams, each of length L and height h. The unit normal $\mathbf{n}$ to ∂B is defined to point away from B (i.e. into a beam).

2.1. Regular BIE—source point approaching a beam surface s_1^+

It is first noted that, for this problem, one can write (see, e.g. [26,33]):

$$\phi(\boldsymbol{\xi}) = -\int_s \frac{\ln r(\boldsymbol{\xi}, \mathbf{y})\sigma(\mathbf{y})\,\mathrm{d}s(\mathbf{y})}{2\pi\varepsilon} + C, \tag{2}$$

where σ is the charge density, per unit area, at a point on a beam surface and s is the total surface of the two beams. Also, $\mathbf{r}(\boldsymbol{\xi}, \mathbf{y}) = \mathbf{y} - \boldsymbol{\xi}$ (with $\boldsymbol{\xi}$ a source and $\mathbf{y}$ a field point), $r = |\mathbf{r}|$, and ε is the dielectric constant of the medium outside the conductors. The constant C is given by the