Engineering Analysis with Boundary Elements 30 (2006) 949–962
ENGINEERING ANALYSIS with BOUNDARY ELEMENTS
www.elsevier.com/locate/enganabound

Analysis of electrostatic MEMS using meshless local Petrov–Galerkin (MLPG) method

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Received 27 January 2006; accepted 14 April 2006
Available online 6 September 2006

Abstract

We analyze electrostatic deformations of rectangular, annular circular, solid circular, and elliptic micro-electromechanical systems (MEMS) by modeling them as elastic membranes. The nonlinear Poisson equation governing their deformations is solved numerically by the meshless local Petrov–Galerkin (MLPG) method. A local symmetric augmented weak formulation of the problem is introduced, and essential boundary conditions are enforced by introducing a set of Lagrange multipliers. The trial functions are constructed by using the moving least-squares approximation, and the test functions are chosen from a space of functions different from the space of trial solutions. The resulting nonlinear system of equations is solved by using the pseudoarclength continuation method. Presently computed values of the pull-in voltage and the maximum pull-in deflection for the rectangular and the circular MEMS are found to agree very well with those available in the literature; results for the elliptic MEMS are new.

Keywords: Micro-electromechanical systems; Pull-in instability; Meshless method; Pseudoarclength continuation method

1. Introduction

Recent technological developments and increasing market demand have opened promising research opportunities and engineering priorities in the field of micro-mechanics. The study of electrostatically actuated micro-electromechanical systems (MEMS) is a special branch of micro-mechanics. These MEMS are widely used in switches, micro-mirrors and micro-resonators. At the microscopic scale, high-energy densities and large forces are available, and the electrostatic actuation may dominate over other kinds of actuation.

Most of the electrostatically actuated MEMS consist of an elastic plate suspended over a stationary rigid plate. The plates are conductive and a dielectric material fills the gap between them. An applied electric voltage between the two plates results in the deflection of the elastic plate, and a consequent change in the MEMS capacitance. The applied electrostatic voltage has an upper limit, beyond which the electrostatic Coulomb force is not balanced by the elastic restoring force in the deformable plate, the two plates snap together and the MEMS collapses. This phenomenon, called pull-in instability, was simultaneously observed experimentally by Taylor [1], and Nathanson et al. [2]. The accurate estimation of the pull-in voltage is crucial in the design of electrostatically actuated MEMS device. In particular, in micro-mirrors [3] and micro-resonators [4] the designer avoids this instability in order to achieve stable motions; on the other hand in switching applications [5] the designer exploits this effect to optimize the performance of the device.

A simple model for estimating the pull-in parameters proposed in [2] is based on a lumped mass–spring system. This model qualitatively describes the pull-in phenomenon but it overestimates the pull-in voltages for many applications [6]. A possible extension of the lumped model consists of modeling the suspended plate as a membrane, and discarding the fringing electric fields. As discussed in [7], the membrane approximation is accurate and reliable for many MEMS devices such as micro-pumps made of thin

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0955-7997/$ - see front matter
doi:10.1016/j.enganabound.2006.04.008

glassy polymers, and grating light valves comprised of stretched thin ribbons. More refined linear and nonlinear models have been studied in [8,9].

Solutions by the shooting method (see e.g. [10, Chapter 7] for a discussion, and a list of references) can be obtained only for particular MEMS geometries that exhibit specific symmetries [11], that allow for the reduction of two-dimensional (2-D) to 1-D problems. However, it has been found in [12] that these simplifications may miss some unstable branches. Indeed, for an annular circular membrane the solution after the pull-in instability may break the symmetry inherited by the domain shape, loading, and boundary conditions. Therefore, it is crucial to develop accurate and reliable numerical methods for determining the pull-in instability parameters, and study, for arbitrary geometries, the MEMS behavior beyond the pull-in instability. Bao and Mukherjee [13] and Chyuan et al. [14] have employed the boundary element method to analyze MEMS.

Recently, considerable research in computational mechanics has been devoted to the development of meshless methods. One objective of these methods is to eliminate, or at least alleviate the difficulty of meshing and remeshing the entire structure, by only adding nodes at or deleting nodes from desired locations in the structure. Meshless methods may also alleviate some other problems associated with the finite element method, such as locking and element distortion. In many applications, they provide smooth and accurate approximate solutions with a reduced number of nodes. Therefore, only a few variables are needed in numerical models.

Meshless methods include the element-free Galerkin [15], hp-clouds [16], the reproducing kernel particle (RPK) [17], the smoothed particle hydrodynamics [18], the diffuse element [19], the partition of unity finite element [20], the natural element [21], meshless Galerkin using radial basis functions [22], the meshless local Petrov–Galerkin (MLPG) [23], and the modified smoothed particle hydrodynamics (MSPH) [24]. All of these methods, except for the MLPG, the MSPH, and the collocation, use shadow elements for evaluating integrals in the governing weak formulations [25].

The MLPG method has been successfully applied to several linear problems in mechanics: static linear plane elasticity [23]; vibrations of elastic planar bodies [26]; static analysis of thin plates [27]; static analysis of beams [28]; vibrations of cracked beams [41]; static analysis of functionally graded materials [29]; analysis of dynamic thermomechanical deformations of functionally graded materials [30]; analysis of axisymmetric transient heat conduction in a bimaterial disk [31]; and wave propagation in a segmented linear elastic bar [32]. Nonlinear problems analyzed with the MLPG method include adiabatic shear banding in thermoviscoelastoplastic materials [33], and the analysis of pull-in instability in micro-beams [34].

In order to completely eliminate a background mesh, the MLPG method is based on a local weak formulation of the governing equations and employs meshless interpolations for both the trial and the test functions. The trial functions are constructed by using the moving least squares (MLS) [35] approximation which relies on the location of scattered points in the body. In the Petrov–Galerkin formulation, test functions may be chosen from a different space than the space of trial solutions. Thus, several variations of the method may be obtained (see e.g. [25] for details).

Here we use the MLPG method to investigate the behavior of electrostatically actuated MEMS modeled as elastic membranes, and the method of Lagrange multipliers to impose displacement-type boundary conditions. Hence, a local symmetric augmented weak formulation (LSAWF) of the problem is introduced. Trial functions in the weak formulation are constructed using the MLS approximation, and weight functions in the MLS framework are chosen as test functions (MLPG1, [25]). In order to find the MEMS deformations beyond the pull-in instability the pseudoarclength continuation method (see e.g. [36,37]) is employed for solving the system of nonlinear equations resulting from the MLPG formulation. The method is applied to four distinct geometries: a rectangle, a circular disk, an annular disk, and an elliptic disk. In the first case, the effect of partial electrodes is studied, and computed results are compared with those obtained with the shooting method applied to the problem derived by generalizing the approach of [7] to partially electroded plates. In the second case, the computed results are validated by comparing them with those of [38]. For the annular circular disk, when symmetry breaking occurs, and no solution by the shooting method is available, the computed solution is compared with the finite-difference solution of [12]. To the authors' knowledge, the elliptic geometry has not been studied thus far. Here, the effect of the ellipse aspect ratio on the pull-in instability of the MEMS is investigated, and MLPG results are compared with those obtained by solving the 2-D boundary-value problem by the finite-difference method.

The rest of the paper is organized as follows. In Section 2 we present governing equations of the electrostatically actuated MEMS. In Section 3 we describe the MLPG method, including the LSWAF, the resulting set of nonlinear equations for the fictitious nodal deflections, and the pseudoarclength method. Computed solutions are presented in Section 4, and comparisons are made with available results. Conclusions are summarized in Section 5. In Appendix A we review the MLS approximation for constructing basis functions, and in Appendix B we give a brief description of confocal elliptic coordinates.

2. Governing equations

A schematic sketch of the problem studied is shown in Fig. 1. We assume that (i) both plates are perfect conductors, and are separated by a dielectric layer of permittivity $\in$, (ii) the bottom plate is rigid, and the top one is flexible, and can be modeled as a membrane, (iii) the membrane is either clamped or free on the