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Non-linear viscoelastic constitutive model for bovine cortical bone tissue

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ABSTRACT

In the paper a constitutive law formulation for bovine cortical bone tissue is presented. The formulation is based on experimental studies performed on bovine cortical bone samples. Bone tissue is regarded as a non-linear viscoelastic material. The constitutive law is derived from the postulated strain energy function. The model captures typical viscoelastic effects, i.e. hysteresis, stress relaxation and rate-dependence. The elastic and rheological constants were identified on the basis of experimental tests, i.e. relaxation tests and monotonic uniaxial tests at three different strain rates, i.e. $\dot{\lambda} = 0.1\,\mathrm{min}^{-1}$, $\dot{\lambda} = 0.5\,\mathrm{min}^{-1}$ and $\dot{\lambda} = 1.0\,\mathrm{min}^{-1}$. In order to numerically validate the constitutive model the fourth-order stiffness tensor was analytically derived and introduced to Abaqus® finite element (FE) software by means of UMAT subroutine. The model was experimentally validated. The validation results show that the derived constitutive law is adequate to model stress–strain behaviour of the considered bone tissue. The constitutive model, although formulated in the strain rate range $\dot{\lambda} = 0.1–1.0\,\mathrm{min}^{-1}$, is also valid for the strain rate values slightly higher than $\dot{\lambda} = 1.0\,\mathrm{min}^{-1}$.

The work presented in the paper proves that the formulated constitutive model is very useful in modelling compressive behaviour of bone under various ranges of load.

1. Introduction

The structure of cortical bone is different than that of trabecular tissue. Also mechanical properties of those tissues, as well as their roles in the human organism, are unlike. However, from macroscopic point of view the damage propagation pattern in both tissues is very similar, i.e. their elastic properties decrease with increase of load [1,2]. This observation was utilised by Garcia et al. [3] who formulated a constitutive law for cortical and trabecular bone simultaneously. The law is phenomenological and macroscopic and is based on three different deformation evolution modes, i.e. pure elastic regime, simultaneous plasticity and damage and pure plastic mode. Because of the fact that the model was to be applied for cortical and trabecular bone the authors defined variables that differentiated them, i.e. volume fraction, responsible for heterogeneity of the tissues, and fabric tensor which defined anisotropy of bone. Damaging behaviour was described by means of a halfspacewise generalisation of the Hill criterion that

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accounts for the distinct damage thresholds in tension and compression of bone tissue. The plastic yield criterion was defined by the fabric-based elastic compliance tensor. One of the restrictions of the model is that damage was defined as a scalar variable affecting equally all constants of the elastic stiffness tensor. This, in consequence, limits the applicability of the model to proportional loading and unloading, which means that the model is capable to cover the hysteresis loop but the loading and unloading parts of stress–strain curves are parallel, which is not true for bone behaviour. Another limitation of the model is that it is rate-independent.

More advanced constitutive model for cortical bone was described in [4]. The authors developed a visco-elasto-plastic formulation for human cortical bone. The experimental validation of the model was conducted utilising literature data obtained from creep and tensile tests [5,6]. Unlike the described two constitutive laws the model presented in [4] is rate dependent. On the basis of FE simulations the authors concluded that it shows a very good capability to describe the tensile behaviour of the cortical bone. The model, however, was formulated only for loading phase of the tensile tests conducted at five strain rates. Thus, it was not capable to capture one of the viscoelastic phenomena, i.e. hysteresis loop. In addition, the stress–strain curves predicted by the model were approximated by piece-wise linear segments. This implied that the model stress–strain curves were not continuous in the regarded strain range, which is not the case in the real bone behaviour.

Cortical bone was thoroughly examined in [7]. The authors formulated a linear constitutive model for the tissue that captured its viescoelastic and viscoplastic behaviour. The viscoelastic effects were modelled by two Maxwell elements and one Hooke's element in parallel. The viscoplastic contribution was modelled by Ramber–Osgood equation. The constants in the model were identified on the basis of data taken from previous experimental studies [8–10]. The main advantage of the formulated constitutive law is that it is valid in a very wide range of strain rate, i.e. from 0.001 s^{-1} to 1500 s^{-1}. The model, however, is valid for small strains and is not able to capture the non-linear hysteresis effects of the tissue. Also, it does not seem to be able to describe another viscoelastic effects, i.e. stress relaxation and creep.

The constitutive law for cortical bone proposed in this paper was derived from the postulated strain energy function. That approach is widely utilised in constitutive model formulation for various materials including biomaterials [11,12]. The model is rate-dependent and takes into account transversal isotropy and compressibility of the bone tissue. It is capable to simulate viscoelastic phenomena of bone, i.e. stress relaxation and hysteresis. The formulation of a non-linear constitutive law, which is based on compressive stress relaxation and monotonic compression tests, is the new contribution to constitutive modelling of bone tissue. The proposed strain energy function is also a novel aspect of the investigations. The number of relaxation times and corresponding characteristic amplitudes was determined utilising stress relaxation tests data. This number was not assumed a priori as it is often done in constitutive investigations. The formulated constitutive model for cortical bone captures elastic and viscoelastic effects. The viscoplastic contribution is not taken into consideration as bone tissue under compressive loads does not indicate viscoplastic characteristics. This will be discussed later. Comparing to constitutive equations for cortical bone described in literature the model developed in this paper captures simultaneously viscoelastic phenomena that take place in bone, i.e. stress relaxation, hysteresis and strain-rate dependency. Another aim of the paper is to present a methodology of constitutive model formulation for bone tissue.

The constitutive research presented here is a continuation of the work shown in Ref. [13] in which a simplified constitutive model for bovine cortical bone is proposed. The model proposed in Ref. [13] is not capable to simulate the hysteresis effect and is not strain-rate dependent. Secondly, the model was not implemented in a FEM software. Thirdly, bone is regarded as an incompressible material. Finally, the number of relaxation times and their values were assumed a priori, which is not the case in this manuscript. All those restraints make the constitutive model for bone considerably limited. The model proposed in the manuscript is highly more advanced than that described in Ref. [13], which is shown below.

2. Materials and method

2.1. New constitutive model formulation

In non-linear viscoelasticity the constitutive model can be formulated by convolution of strain-dependent function $\mathbf{S}^e(\lambda)$ and a time-dependent function $g(t)$:

$$\mathbf{S}(\lambda, t) = \mathbf{S}^e(\lambda) * g(t), \tag{1}$$

where $\mathbf{S}$ is the second Piola–Kirchhoff stress tensor, $\mathbf{S}^e$ is the elastic second Piola–Kirchhoff stress tensor, $g(t) = g_\infty + \sum_{i=1}^{n} g_i \cdot \exp(-t/\tau_i)$, $g_\infty = 1 - \sum_{i=1}^{n} g_i$, g_i are characteristic amplitudes and τ_i is the relaxation times, λ is stretch ratio along the loading direction, t represents time. The general constitutive Eq. (1) expresses stress as a function of strain and time, which makes the model viscoelastic and non-linear. Eq. (1) can be written in the form:

$$\mathbf{S}(t) = g_\infty \mathbf{S}^e(t) + \sum_{i=1}^{n} \int_0^t g_i \cdot \exp\left(-\frac{t-s}{\tau_i}\right) \cdot \frac{\partial \mathbf{S}^e(s)}{\partial s}\, ds, \tag{2}$$

where s represents the historical time variable. The elastic part of the second Piola–Kirchhoff stress is calculated using the formula:

$$S_{ij}^e = 2 \cdot \frac{\partial \Psi_e}{\partial C_{ij}}, \tag{3}$$

where C_{ij}, (i, j = 1, 2, 3) are components of the right Cauchy deformation tensor $\mathbf{C}$.

The strain energy function Ψ_e can be written in the general form (see e.g. [14]):

$$\Psi_e = \overline{\Psi}_e + \hat{\Psi}_e, \tag{4}$$

where $\overline{\Psi}_e$ and $\hat{\Psi}_e$ represent the isochoric and volumetric parts of the function Ψ_e, respectively. The new form of $\overline{\Psi}_e$ proposed in this paper is as follows:

$$\overline{\Psi}_e = \frac{c_1}{2}(\bar{I}_1 - 3) + \frac{c_2}{2c_3}(\bar{I}_1^{c_3} - 3^{c_3}) + \frac{c_4}{2}(\bar{I}_4 - 1)\ln \bar{I}_4, \tag{5}$$

where $\bar{I}_1$ and $\bar{I}_4$ are the first and fourth invariants of tensor $\overline{\mathbf{C}} = J^{-2/3}\mathbf{C}$, J is the determinant of the deformation gradient