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A wave finite element-based approach for the modeling of periodic structures with local perturbations

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ABSTRACT

The wave finite element (WFE) method is investigated to describe the dynamic behavior of finite-length periodic structures with local perturbations. The structures under concern are made up of identical substructures along a certain straight direction, but also contain several perturbed substructures whose material and geometric characteristics undergo arbitrary slight variations. Those substructures are described through finite element (FE) models in time-harmonic elasticity. Emphasis is on the development of a numerical tool which is fast and accurate for computing the related forced responses. To achieve this task, a model reduction technique is proposed which involves partitioning a whole periodic structure into one central structure surrounded by two unperturbed substructures, and considering perturbed parts which are composed of perturbed substructures surrounded by two unperturbed ones. In doing so, a few wave modes are only required for modeling the central periodic structure, outside the perturbed parts. For forced response computation purpose, a reduced wave-based matrix formulation is established which follows from the consideration of transfer matrices between the right and left sides of the perturbed parts. Numerical experiments are carried out on a periodic 2D structure with one or two perturbed substructures to validate the proposed approach in comparison with the FE method. Also, Monte Carlo (MC) simulations are performed with a view to assessing the sensitivity of a purely periodic structure to the occurrence of arbitrarily located perturbations. A strategy is finally proposed for improving the robustness of periodic structures. It involves artificially adding several "controlled" perturbations for lowering the sensitivity of the dynamic response to the occurrence of other uncontrolled perturbations.

1. Introduction

The problem of predicting the dynamic behavior of finite-length periodic structures with local perturbations is tackled in the present paper. Such systems are composed, along a certain straight direction, of identical substructures which can be of arbitrary shape, e.g., like an aircraft fuselage. Assessing the sensitivity of the frequency response of those structures to the occurrence of small perturbations which might be arbitrarily located — e.g., such as variabilities of the design processes, or defects — is the motivation behind the present study. The challenge concerns the development of a numerical approach which is low time-consuming in comparison with the conventional FE method, while keeping a high level of accuracy. Time-harmonic elasticity problems are considered here. Among the wave-based numerical approaches, the WFE method has proved to be relevant for modeling purely periodic structures, and will be thus improved in this work with a view to modeling periodic structures with local perturbations.

Originally, the WFE method has been developed to describe the wave propagation along one-dimensional periodic structures [1–3]. The procedure is nothing else but a transfer-matrix approach and the use of Bloch's theorem [4] for expressing the so-called wave modes, i.e., waves which propagate from substructure to substructure along the right and left directions of a periodic structure. To date, the WFE method has been applied to homogeneous waveguides, such as beams, plates, multi-layered systems and fluid-filled pipes [5–9], simple periodic structures [10,11] or more complex systems such as tires [12], cochlea [13] and metamaterials [14]. Also, it has been applied in vibroacoustics to analyze thick layered panels [15] and poroelastic media [16]. Finally, in [17], a formulation of the scattering matrix for guided acoustical propagation, based on the FE and WFE methods, has been

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proposed; also, in [18], the accuracy of the WFE method has been analyzed.

The forced response of finite-length periodic structures has been analyzed in various ways by means of the WFE method [19–22]. The strategy consists in expanding vectors of displacements and forces in bases of wave modes. For instance, the study of assemblies involving elastic waveguides and junctions has been proposed in [23], while that of truly periodic structures — i.e., which are made up of heterogeneous substructures — has been investigated in [24]. More recently, a model reduction technique has been proposed for the study of the forced response of periodic structures made up of arbitrary-shaped substructures which are modeled with many degrees of freedom (DOFs) [25]. The strategy consists in partitioning a given periodic structure into one central structure which is modeled by means of the WFE method, and two surrounding substructures which are modeled with the FE method. In doing so, it becomes possible to model the central structure with a small number of wave modes only. This is explained by the fact that the kinematic and mechanical fields admit smooth variations on the boundaries of the central structure — i.e., the interfaces between the central structure and the surrounding FE-based substructures. In other words, a small number of wave modes are only needed to accurately capture the boundary conditions (BCs) of the central structure. This strategy has been proved to be relevant, e.g., for computing the forced response of a periodic structure whose substructures are described with more than 30,000 DOFs [25]. The remarkable feature of the approach is that it can be easily implemented on MATLAB®, and yields small computational times when compared to dedicated FE softwares.

The dynamic behavior of periodic structures which contain several perturbed substructures — which can be different from each other — is analyzed throughout the present paper by means of the WFE method. Within the present scope, those perturbed substructures undergo arbitrary variations of their material and geometric characteristics, and can be located in arbitrary way along a periodic structure. The motivation lies in the development of a numerical tool which is fast and accurate for computing the forced response of these systems. Such an analysis appears to be completely original and has never been carried out previously. Here, original improvements of the reduction technique developed in [25] are proposed which involve introducing perturbed parts which are built from a FE-based perturbed substructure and two surrounding FE-based unperturbed ones. In doing so, one expects smooth variations of the kinematic and mechanical fields on the left and right sides of each perturbed part, i.e., they can be described with a small number of wave modes. In addition, a new reduced wave-based matrix formulation is developed for the computation of the forced response of perturbed periodic structures, and which follows from the consideration of small-sized transfer matrices between the right and left sides of the perturbed parts.

The rest of the paper is organized as follows. In Section 2, the basics of the WFE method are recalled. The numerical strategy used to compute the wave modes which travel along a periodic structure made up of arbitrary-shaped substructures is presented. Also, the strategy to compute the forced response of periodic structures is detailed. In Section 3, the forced response of periodic structures with perturbed substructures is analyzed. The aforementioned model reduction technique — which consists in partitioning a whole periodic structure into one central structure surrounded by two FE-based extra substructures, and considering FE-based perturbed parts which are built from a perturbed substructure and two surrounding unperturbed ones — is developed. The derivation of the transfer matrices of the perturbed parts is detailed, along with the wave-based matrix formulation to compute the forced response of periodic structures with local perturbations. In Section 4, numerical experiments are carried out in order to validate the proposed approach in comparison with the conventional FE method. Also, Monte Carlo (MC) simulations are performed to assess the sensitivity of a purely periodic structure to the occurrence of arbitrarily located perturbed substructures. A strategy is finally proposed for improving the robustness of periodic structures. In this framework, several "controlled" perturbed substructures are artificially added for reducing the sensitivity of the dynamic response to the occurrence of other uncontrolled perturbations.

2. WFE method

2.1. Mechanical problem

The approach proposed in this work can be applied to different kinds of time-harmonic linear mechanical problems. To fix ideas, consider a two-dimensional plane stress linear elastic problem consisting in finding a displacement vector field $\mathbf{u}(x,y)$ on an open bounded domain Ω. In this case, the elastodynamic equation writes

$$\operatorname{div}\boldsymbol{\sigma} = -\rho\omega^2\mathbf{u} \quad \text{on } \Omega, \tag{1}$$

where ρ is the density, ω is the angular frequency and $\boldsymbol{\sigma}$ is the stress tensor defined by

$$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\epsilon}, \tag{2}$$

where $\boldsymbol{\epsilon}$ is the strain tensor and $\mathbf{C}$ is the fourth-order stiffness tensor. Using engineering notations, Eq. (2) can also be written as

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix}\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix}, \tag{3}$$

where E is Young's modulus and ν is Poisson's coefficient. Assume that the boundary of Ω — namely, Γ — is sufficiently regular and is split into two parts Γ_u and Γ_t such that $\Gamma = \Gamma_u \cup \Gamma_t$ and $\Gamma_u \cap \Gamma_t = \varnothing$, and defined so that

$$\begin{aligned} &\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_u, \\ &\mathbf{t} = \mathbf{t}_0 \quad \text{on } \Gamma_t, \end{aligned} \tag{4}$$

where $\mathbf{t} = \boldsymbol{\sigma}\cdot\mathbf{n}$ is the traction vector on Γ, $\mathbf{n}$ is the unit outward normal vector, and $\mathbf{t}_0$ is a given vector-valued function. For the sake of simplicity, only a homogeneous displacement condition is considered on Γ_u. Notice however that a non-homogeneous displacement condition may be considered without difficulty provided the following relations are slightly adapted.

The aforementioned problem can be classically solved from the variational method. In this framework, the elastodynamic equation (1) is multiplied by a test function $\mathbf{v}(x,y)$ such that $\mathbf{v} = \mathbf{0}$ on Γ_u, as follows:

$$\int_\Omega \mathbf{v}\cdot\operatorname{div}\boldsymbol{\sigma}\,dx = -\int_\Omega \rho\omega^2\mathbf{v}\cdot\mathbf{u}\,dx. \tag{5}$$

Integrating by part the first integral yields

$$-\int_\Omega \boldsymbol{\epsilon}(\mathbf{v})\cdot\mathbf{C}\cdot\boldsymbol{\epsilon}(\mathbf{u})\,dx + \int_{\Gamma_t}\mathbf{v}\cdot\mathbf{t}_0\,ds = -\int_\Omega \rho\omega^2\mathbf{v}\cdot\mathbf{u}\,dx. \tag{6}$$

Following the Galerkin method for finite elements, approximate solutions are sought as

$$\begin{aligned} &\mathbf{u}(x,y) \approx \mathbf{N}(x,y)\mathbf{q}, \\ &\mathbf{v}(x,y) \approx \mathbf{N}(x,y)\boldsymbol{\delta}\mathbf{q}, \end{aligned} \tag{7}$$

where $\mathbf{N}$ is the so-called matrix of shape functions, while $\mathbf{q}$ and $\boldsymbol{\delta}\mathbf{q}$ are vectors of nodal displacements. Inserting Eq. (7) into the