Finite Elements in Analysis and Design 119 (2016) 15–29

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Full Length Articles

A reduced basis approach for the parametric low frequency response of submerged viscoelastic structures

C. Leblond *, J.-F. Sigrist

DCNS Research, Département Dynamique des Structures, 5 rue de l'Halbrane, 44340 Bouguenais, France

ARTICLE INFO

Article history:
Received 1 September 2015
Received in revised form
23 March 2016
Accepted 16 May 2016
Available online 3 June 2016

Keywords:
Structure–acoustic interaction
Viscoelasticity
Finite element method
Reduced order model
Reduced basis
Greedy algorithm

ABSTRACT

Reduced Order Models (ROMs) are proposed to compute the frequency response of submerged viscoelastic structures with variation of parameters. A parameterized matrix system is first derived from the finite element discretization of a structural-acoustic formulation, to constitute both the full model of reference and the starting point of the ROMs building. The low dimensional trial subspace is obtained offline with an iterative greedy algorithm inspired by the Reduced Basis method. Galerkin and Minimum Residual projections are then considered to determine the test subspace and construct the parametric online-efficient ROMs. Their accuracy and robustness are evaluated on two test cases: a submerged composite viscoelastic plate and a submerged propeller with viscoelastic patches. They are moreover compared to established methods conceived to tackle in vacuo viscoelastic problems, and extended here for submerged structures. Convergence properties as functions of the decomposition order are computed and discussed on parameter spaces of dimensions one and four. The resulting CPU time gain compared to full model evaluations opens the way to efficient design strategies, taking into account the parameters variabilities.

1. Introduction

The computation of submerged structures responses to weak forced mechanical solicitations remains a CPU time-consuming process in naval industry, on account on the large size of the underlying problems, their geometrical complexities, as well as the strong coupling with the fluid. The finite element discretization of such a forced vibration problem usually results in a linear matrix system with millions of degrees of freedom, which has to be inverted at each frequency of interest. In practice, analyzes on very coarse frequency grids are performed in order to reduce the overall time of the computation. In the design process of vessels or submarines, the choice of the materials and their characteristics, if known, may moreover evolve, so that numerous computations are theoretically required with new values of the parameters. Once again, the large CPU-time requirement drastically limits the number of shots performed in practice, which may ultimately affect the robustness and optimality of the design.

In order to reduce both the structural vibrations and the noise radiation, viscoelastic damping technologies are receiving a growing interest in naval industry. The use of such highly dissipative materials nevertheless complicates the numerical modeling and computation of the problem. For instance, the stiffness matrix resulting from the finite element discretization becomes complex-valued and frequency-dependent. Furthermore, viscoelastic materials are very sensitive to non-mastered operational conditions, for example the temperature, and their constitutive mathematical model are rarely fully determined. A robust design should take into account these uncertainties, increasing the dimension of the parameter space to consider.

Model reduction techniques are developed and evaluated in this paper, so as to make tractable analyzes on very fine frequency grids, as well as to handle the complexity of a relatively high-dimensional parameter space, for viscoelastic and submerged structures. Reduced Order Models (ROMs) are nowadays able to represent complex systems with few degrees of freedom at the cost of a moderate loss of accuracy. Numerous techniques can be found in the literature to build a ROM. Most of them involve the knowledge of a low-dimensional basis as starting point, such as Proper Orthogonal Decomposition [1], balanced Proper Orthogonal Decomposition [2], A Priori Reduction [3,4], Proper Generalized Decomposition [5–9] and Reduced Basis [10,11], just to name a few. The crucial point to obtain a reliable parametric ROM is to build a reduced trial space that spans most of the physics at hand, over the whole parameter space of interest. Obviously, the reduced basis has to be of much lower dimension than the discretized full model, for a real-time (or at least very fast) online evaluation of the ROM solution.

* Corresponding author. Tel.: +33 2 44 76 57 93.
E-mail address: cedric.leblond@dcnsgroup.com (C. Leblond).

http://dx.doi.org/10.1016/j.finel.2016.05.002

In the case of a one-dimensional parameter space, involving only the frequency ω, established reduction methods can be found in the literature to tackle part of the problem of interest (a submerged viscoelastic structure):

- For linear problems involving *in vacuo* elastic structures, the classical modal basis remains the most appropriate basis for reduction. The real-valued eigenvectors are computed through highly efficient and powerful algorithms, even for very large sparse symmetrical real-valued mass and stiffness matrices. The Proper Orthogonal Decomposition and A Priori Reduction have also been evaluated with success for the reduction of such linear (and nonlinear) structural dynamics problems [12,13].
- For *in vacuo* viscoelastic structures, the discretized full model equation can take the form $\left[-\omega^2\mathbf{M}+\mathbf{K}(\omega)\right]\mathbf{X}(\omega)=\mathbf{B}(\omega)$, with the symmetrical real-valued matrix $\mathbf{M}$ and the symmetrical complex-valued and frequency-dependent matrix $\mathbf{K}(\omega)$. The corresponding eigenvalue problem, $\left[-\omega^2\mathbf{M}+\mathbf{K}(\omega)\right]\Psi=0$, appears to be non-trivial due to its non-linearity. The Modal Strain Energy (MSE) approach [14] consists in neglecting the frequency dependence of the stiffness matrix to obtain the eigenvectors: they are therefore solutions of the eigenvalue problem $\left[-\omega^2\mathbf{M}+\mathbf{K}(\omega_0)\right]\Psi=0$, with ω_0 being the minimum frequency of interest for instance. To reduce the computational time of the basis calculation and work only with real-valued modes, a widespread practice consists in keeping only the real part of the stiffness matrix when solving this eigenvalue problem. These approaches, easy to implement and cost-effective, are of course limited to weakly damped materials or to a close vicinity of ω_0. For highly damped structures, more expensive and accurate methods are available. For instance with the Iterative Modal Strain Energy (IMSE) approach [15], each mode is computed through an iterative algorithm, in which the real part of the stiffness matrix is updated in the eigenvalue problem $\left[-(\omega^{(i)})^2\mathbf{M}+Re\{\mathbf{K}(\omega_m^{(i-1)})\}\right]\Psi^{(i)}=0$. The Iterative Complex Eigensolution (ICE) approach [16,17] follows the same iterative strategy, while keeping the whole complex stiffness matrix. It is therefore more physically consistent for highly damped structures, but also more time-consuming due to less powerful algorithms available to solve large complex eigenvalue problems. Another strategy consists in computing several bases, solutions of the eigenvalue problems $[-\omega^2\mathbf{M}+\mathbf{K}(\omega_j)]\Psi=0$ for $j=1\ldots J$, and in assembling them with a Gram–Schmidt process. The choice of the values of ω_j is performed *ad hoc*, guided by the physics of the problem. This multi-model approach has been successfully evaluated on highly damped structures [18,19]. An alternative family of strategies, applied on problems involving viscoelasticity and nonlinear behavior [20], considers the frequency as an extra-coordinate. This approach based on Proper Generalized Decomposition leads to efficient and accurate harmonic analysis [21,22]. It has been successfully applied on several test cases, involving relatively light full scale models.
- For elastic and weakly viscoelastic structures submerged in a finite fluid domain, the available methods are mostly based on a dynamic substructuring procedure [23], involving the acoustic modes and the zero-frequency static solutions in the fluid part, as well as the undamped and uncoupled structural modes. These potentially efficient reduction approaches require a development effort to be industrially usable, since they are not based on a precomputed monolithic structural-acoustic full model. Another strategy consists in using the boundary elements method to account for the infinite fluid domain effect [24], through a frequency-dependent additional mass matrix in the structural finite element matrix system. The reduction may then be achieved via a Galerkin projection onto the in vacuo undamped structural modes [24–27].

This review, although not comprehensive, suggests that the literature on the space-frequency reduction of problems involving submerged viscoelastic structures is rather scarce: their accuracies may be guaranteed for weakly damped structures, but the question of robustness of the reduced trial basis remains open for highly damped materials. The most suitable and available methods for industrial use still involve *ad hoc* and non-automatic procedures, at the risk of inaccurate estimations of the quantities of interest.

The multidimensional character of the parameter space constitutes here an additional challenge. This aspect does not appear to be specifically tackled in the field of structure–acoustic interaction problems, despite underlying industrial needs. Several techniques are of course emerging in other domains to treat such parametric studies. Among them, a line of research focuses on the interpolation of precomputed bases, to yield a space basis adapted to a new parameters values. More precisely, the subspace angle interpolation [28], as well as an advanced interpolation based on the Grassmann manifold and its tangent space [29], are developed and successfully tested in the field of aeroelasticity. Since these approaches are essentially geometrical and do not take into account the underlying equations, the precomputed bases still require to be close enough to the new parameters values to yield an accurate ROM. The Proper Generalized Decomposition (PGD) constitutes another serious candidate, with numerous successful applications on multidimensional parametric models [30–32]. More precisely, the family of strategies considering the frequency as an extra-coordinate has also been developed and evaluated in the parametric framework [21,22]. It is shown that, with the increase of dimensionality, the internal fixed point algorithm requires more iterations to converge and that the PGD expansion requires a higher rank to reach a given accuracy. The overall number of calls to the full scale model is nevertheless reasonable, which enables us to decrease by several orders of magnitude the CPU time compared to a brute force approach sampling. The application of a PGD-like approach for the computation of the submerged structures responses with uncertain parameters is indeed a work in progress [33,34]. We address in the present study the challenge of sampling a relatively high-dimensional parameter space to iteratively build an appropriate trial basis, by following the framework of the Reduced Basis method along with the use of a greedy algorithm [10,11,35,36]. Contrary to PGD algorithms, there is no fixed point algorithm to compute each new term of the expansion; the overall number of calls to the full scale model is therefore expected to be very small, which is of paramount importance for industrial applications.

The goal is here to obtain a parametric ROM, suitable for real-time online use and accurate when the parameters vary. The offline computation of the reduced basis may be CPU time-consuming, but it has to be of the same order of a standard industrial study: in other words, it should be feasible on a workstation for scientific computation in a design office. Another constraint of the study lies on the weakly intrusiveness of the reduction method: the fluid–structure full scale model, on which the reduced model is built, does not have to be fully considered as a black-box, but the effort of implementation within it has to remain limited. More precisely, the approach is conceived to be usable within a standard industrial FEM software, by definition less opened than a homemade dedicated numerical code. The resulting reduction strategy proposed here follows this guideline.