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A laminated structural finite element for the behavior of large non-linear reinforced concrete structures

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ABSTRACT

In order to correctly predict the kinematics of complex structures, analysis using three-dimensional finite elements (3DFEs) seems to be the best alternative. However, simulation of large multi-layered structures with many plies can be unaffordable with 3DFEs because of the excessive computational cost, especially for non-linear materials. In addition, the discretization of very thin layers can lead to highly distorted FEs carrying numerical issues, therefore, reduced models arise as an affordable solution.

This paper describes a new finite element formulation to perform numerical simulations of laminated reinforced concrete structures. The intention of this work is that the proposed scheme can be applied in the analysis of *real-life* structures where a high amount of computational resources are needed to fulfill the meshing requirements, hence the resulting formulation has to be a compromise between simplicity and efficiency.

So that, the condensation of a dimension (thickness), mandatory to model three-dimensional structures with two-dimensional finite elements (2DFEs), leads to refer all layers contained within such FEs to a plane, which is typically named *middle* plane or *geometrical* plane, since its sole function is to serve as a geometrical reference. This work is based on the assumption that the *geometrical* plane has to be distinguished from a *mechanical* plane, which is where the resultant stiffness of all layers is contained. It is also assumed in this work that the *mechanical* plane changes its position due to non-linear response of the component materials.

1. Introduction

Current theories that allow the use of two-dimensional FEs to model composite materials, yet powerful, lack the necessary simplicity for their application in complex structures where a large amount of FE is required for a good approximation in the obtained results.

Thus, simpler and more efficient techniques are required for modelling laminated structures, where the three-dimensional description can be reduced to a two-dimensional model by introducing hypotheses on the displacements and/or on the stresses field, since laminate thickness is at least one order of magnitude lower than in-plane dimensions.

Reference [1] provides an overview of the available theories and finite elements that have been developed for multi-layered, anisotropic, composite plate and shell structures. Multi-scale approaches [2,3] can also be used to model non-linear multi-layered materials. In this method a macroscopic model is used to obtain the global response of the structure whereas the material behavior, modelled with a constitutive law, is solved with a microscopic model.

Many reduced approaches have been developed and improved since 19th century. In order to facilitate their classification, they could be distinguished according to [4]:

(a) The type of unknown variable chosen, so they could be *Displacement Based Theories* (DB), *Stress-Based Theories* (SB), or if both stress and displacement are considered as unknowns, a *Mixed Approach* (MB) is obtained.
(b) How the unknown variables are described. In this classification it may be an *Equivalent Single Layer* (ESL) description, where governing equations are written for the whole plate, or

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- a *Layer-Wise* (LW) description, where each layer is treated independently assuming separate displacement/stress fields within each ply, which leads to write the governing equations for each layer.

From the previous classification, the most basic DB-ESL model is the Classical Theory (CT) [5], whereas an improvement to the CT theory is the *First Order Shear Deformation Theory* (FSDT) [6] which enhances the CT kinematics by adding shear effects. Although CT and FSDT are excellent alternatives to accurately model homogeneous thin and thick structures, they lead to poor prediction in the cases listed below

- Where component materials have a high level of transverse anisotropy.
- When applied to the analysis of composite laminated with embedded debounding.
- When it is necessary to provide regions with 3-D stresses fields, i.e. $\sigma_Z \neq 0$.
- When it is required to capture the so called *zig-zag* pattern of in-plane displacements (ZZ condition).
- When it is required to satisfy the condition of continuous transverse shear along the thickness direction (TC condition).

The cause is found in the linear thickness distribution of the axial displacement, which does not match the *ZZ* pattern depicted in Fig. 1 [4].

In order to fulfill the previously listed condition it must be necessary to use either a theory based on 3-D kinematics, or a LW based theory. Although LW theories accurately fulfill both, the ZZ and the TC condition, the number of unknown variables is proportional to the number of analyzed layers. As a result, these models yield not only a high level of accuracy but also to an amount of unknown variables similar to the 3D analysis. For this reason, LW models may result unattractive for simulating large laminated structures with many plies. Therefore, these models should be employed to analyze complex problems where other less expensive approaches fail to give realistic predictions [4].

A special case of LW model where the number of unknowns is independent of the number of analyzed layer is the Zigzag theory (ZZT), which is a good compromise between the accuracy of MB-LW theories and the computational efficiency of DB-ESL models. One of the most important advantages of these theories is that the number of kinematics unknowns is independent of the number of analyzed layers.

Have to be remarked that the study of the cases where it is mandatory to use a LW description is out of the scope of this work, and the only feasible solution from a computational point of view, which allows to achieve good results, is to adopt a ESL scheme.

That is why it has been proposed a scheme capable of reproducing the bending damage of a laminated material without the need of additional degrees of freedom than the ones listed below:

$$\boldsymbol{d} = \{w_1\ \theta_{x1}\ \theta_{y1}\ w_2\ \theta_{x2}\ \theta_{y2}\ w_3\ \theta_{x3}\ \theta_{y3}\}^{\mathrm{T}} \tag{1}$$

This simplification is justified by the fact that stiffness of the simple materials used for reinforced concrete (RC) structures never exceeds an order of magnitude. In addition, in order to avoid shear locking situations [7], the proposed scheme has been implemented using a Discrete Kirchhoff Triangle [8] where the shear transverse strains are postulated to be neglected with respect to other strains.

The paper is organized as follows. We start by presenting the basic framework of the plate theory, and later the proposed modification, to finally detail an implementation into the FE framework. Section 3 describes the application of the proposed scheme to non-linear materials, and consequently, to plates with bending degradation. Finally, in Section 4 it is shown the performance of the proposed plate scheme, and is presented some numerical examples; in particular, a linear clamped beam, a linear clamped plate with a notch, a non-linear clamped beam, an unreinforced concrete frame and a reinforced concrete frame.

2. Basic framework and FE implementation

2.1. Geometry and load

The plate term is referred to a flat slender body, occupying the domain

$$\Omega = \left\{ (x,y,z) \in \mathbb{R} \mid z \in \left[-\frac{t}{2}, +\frac{t}{2} \right],\ (x,y) \in \mathcal{A} \subset \mathbb{R}^2 \right\} \tag{2}$$

where the plane $Z=0$ coincides with the mid-plane (also referred as the geometrical along this work) of the undeformed plate and the transverse dimension, of *thickness* t, is small compared with the other two dimensions.

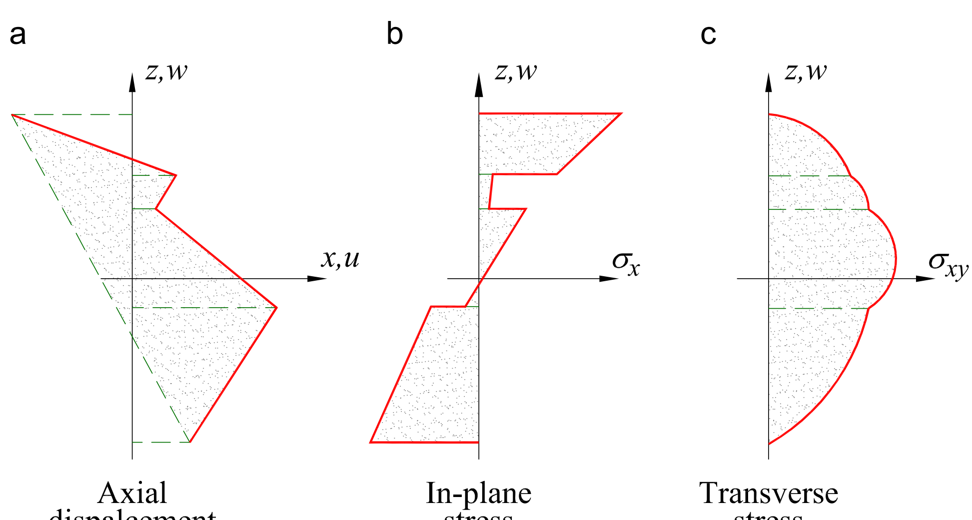

Fig. 1. Continuous zigzag in-plane displacement (a), discontinuous in-plane stress (b), and continuous transverse stress (c).