Finite Elements in Analysis and Design 115 (2016) 33–42

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Solving elastoplasticity problems by the Asymptotic Numerical Method: Influence of the parameterizations

Abdellah Hamdaoui, Bouazza Braikat*, Noureddine Damil

Laboratoire d'Ingénierie et Matériaux LIMAT, Faculté des Sciences Ben M'Sik, Université Hassan II de Casablanca, B.P. 7955 Sidi Othmane, Casablanca, Morocco

ARTICLE INFO

Article history:
Received 5 November 2015
Received in revised form
26 February 2016
Accepted 12 March 2016
Available online 29 March 2016

Keywords:
Regularization technique
Asymptotic Numerical Method
Finite elements
Elastoplasticity
Step length

ABSTRACT

In this paper, we will introduce and discuss new parameterizations to solve elastoplasticity problems by using the Asymptotic Numerical Method (ANM). The elastic–plastic transition and the elastic unloading are taken into account by using the regularization technique proposed in Assidi et al. (2009) [1] and Zahrouni et al. (2005) [2]. The ANM is a family of algorithms for path following problems; each ANM step is based on the computation of truncated vectorial series with respect to a path parameter "a" (Cochelin et al., 1994 [3]). We present and discuss different parameterizations in ANM algorithm for solving elastoplasticity problems, namely the definition of the path parameter "a"; two concepts of parameterization are introduced and compared: a Riks type parameterization which is a combination of both load parameter and time and a parameterization based on the minimization of a rest (Mottaqui et al., 2010 [4,5]). We will also discuss and compare the definitions of the step length in the case of elastoplasticity. Aiming to analyze the quality of the solutions, we will compute and study the residue of all the equations for different values of tolerance parameters of the ANM continuation. To illustrate the performance of these proposed parameterizations and step length definitions, we will give numerical comparisons on structural elastoplasticity problems with the Newton–Raphson method.

1. Introduction

In this paper, we investigate a new procedure to solve the problems of structural mechanics with an elastic–plastic constitutive law, where the equations can be written as:

$$R\left(U(t), \frac{dU(t)}{dt}, C(t)\right) = 0 \quad (1)$$

where R is the so-called residual vector, $U(t)$ is the unknown vector and $C(t)$ is a time-dependent scalar loading parameter and t indicates the time parameter . The computation of solution paths of nonlinear problem (1) are generally done by predictor–corrector algorithms. The most widely used techniques are those of Newton types [6–10].

The chosen algorithm to solve the nonlinear problem (1) belongs to the family of Asymptotic Numerical Methods ANM [3,11]. Classically, this consists of expanding the unknown $U(t)$ and $C(t)$ of the nonlinear problem (1) in power series with respect to a path parameter "a". As the problem (1) is time dependent, we suggest in this work to consider the path parameter "a" as a time function. In Assidi et al. [1], the path parameter has been chosen equal to the time t. Within this framework and using the identity $\frac{dU}{dt} = \frac{dU}{da}\frac{da}{dt}$, each step length of the continuation method is represented by a truncated power series at order N as follows:

$$\left.\begin{aligned} U(a) &= U_0 + \sum_{k=1}^{k=N} a^k U_k \\ C(a) &= C_0 + \sum_{k=1}^{k=N} a^k C_k \end{aligned}\right\} a \in [0, a_{max}] \quad (2)$$

The path parameter shall be defined as a time function using two concepts. (U_0, C_0) is a known and regular solution corresponding to $a=0$ and N is the truncated order of the series. The terms of the series (2) are solutions of a family of well-posed linear problems. After a finite element discretization, these problems can be solved recursively by decomposing only one tangent stiffness matrix by step length. The validity range $[0, a_{max}]$ is deduced from the computation of the truncated vectorial series, each step length can be a posteriori defined using convergence properties of the series which lead to naturally adaptive step length algorithm. The step lengths a_{max} are computed a posteriori by the two following

* Corresponding author.
E-mail address: b.braikat@gmail.com (B. Braikat).

http://dx.doi.org/10.1016/j.finel.2016.03.001
0168-874X/

estimations [11]:

$$a_{maxd} = \left(\epsilon_d \frac{\| U_1 \|}{\| U_N \|}\right)^{\frac{1}{N-1}}, \quad a_{maxr} = \left(\epsilon_r \frac{1}{\| F^{nl}_{N+1} \|}\right)^{\frac{1}{N+1}} \tag{3}$$

where ϵ_d and ϵ_r are given tolerance parameters, U_k is the unknown vector at order k and F^{nl}_{N+1} are the ANM right-hand sides at order $N+1$. In Assidi et al. [1], we have used as step length the minimum of the step lengths (3)-a computed from the series of each component of the unknown vector U. The influence of the tolerance parameters ϵ_d and ϵ_r on the quality of the solution is analyzed by studying the evolution norm of the residual vector R versus the time parameter.

The ANM, which supposes an analytical representation of the response curve, cannot be applied directly to non-regular problems for which governing equations are not analytical functions of the unknowns.

The method of solving non-regular problems has shown its efficiency for unilateral contact problems [12], for plastic behavior within deformation theory of plasticity [13], for viscoplastic laws with Norton–Hoff model [14] and for punching problems coupling several nonlinearities [15].

A first study, which consists in employing a continuation procedure, based on ANM to solve the plasticity problems, has been recently proposed in [1]. A first idea of this algorithm has been proposed by the authors for the one-dimensional case in [2]. The elastic–plastic behavior exhibits two states, the elastic state and the plastic state, the elastic–plastic transition depends only on the stress state whereas the elastic unloading implies the use of the time derivative of these stresses [16]. To define a single regular constitutive relation that takes into account both transitions, we have regularized the two corresponding unilateral conditions. The key point is to establish a regular relation between two scalars: the yield function and the plastic multiplier. The non-regular problem is replaced by the obtained regular problem of Prandtl–Reuss. The efficiency of the proposed continuation method has been shown throughout several tests within structural mechanics with plane stress analysis and by using finite element discretization [1,2].

The main difficulty in view to apply the ANM for elastoplastic structures is the definition of parameterizations of the ANM curves, i.e. the definition of the path parameter a, the definition of the step length and when the considered problem is singular. The present paper is focused on these two first points. The singularity problem is solved in the work of Assidi et al. [1].

We present and discuss different parameterizations in ANM algorithm for solving elastoplasticity problems, namely the definition of the path parameter "a". In Assidi et al. [1] and in Zahrouni et al. [2], the parameterization has been defined as $a=t$ and the step length has been defined as the minimum of a_{maxd} of the components of the unknown vector U. In this work, we suggest using the parameterizations as those defined in [4,5]. Two parameterization concepts will be used. The first concept defines the parameter "a" as a sort of Riks parameterizations [6] between the time-dependent scalar loading parameter $C(t)$ and the time parameter t. The second parameterization will be based on the concept of the minimization of a rest [4,17,18]. In this case no auxiliary equation is used.

We will also discuss and compare the definitions (3) of the step length in the case of elastoplasticity. To illustrate the performance of these proposed parameterizations and step length definitions, some numerical comparisons on structural elastoplasticity problems will be given.

2. Regularized model

The aspects concerning the regularization of an elastoplastic problem are given in Appendix A. Full details are outlined in references [1,2]. We consider an elastoplastic solid occupying a domain Ω and subjected to an external load $C(t)F$ on the boundary $\partial\Omega$; where F is a given vector and $C(t)$ is a time-dependent load parameter. The equilibrium equation can be written as follows:

$$\int_\Omega \sigma : \delta\epsilon \, d\Omega = C(t) \int_{\partial\Omega} F\delta u \, ds \tag{4}$$

where σ and ϵ indicate respectively the Cauchy stress tensor and the strain tensor and u is the displacement. The equations of the regularized elastoplastic model of Prandtl–Reuss in small deformation (see Appendix A) which takes into account the elastic unloading in the 3D case [1] can be written as:

$$\begin{aligned}
&\dot\sigma = D : (\dot\epsilon - \dot\epsilon^p)\\
&\dot\epsilon^p = \dot\lambda n\\
&qn = \frac{3}{2}\sigma^d\\
&q^2 = \frac{3}{2}\sigma^d : \sigma^d + \eta_4^2 \sigma_y^2\\
&Z = GH\\
&\dot\lambda = \dot\epsilon_c Z\\
&H(H-\xi) = \eta_2^2\\
&\dot\epsilon_c \xi = n : \dot\epsilon\\
&G.Den = \eta_1\\
&Den = \frac{\sigma_e}{2\mu}F + \eta_1\left(\frac{3}{2} + \frac{h}{2\mu}(1+f)\right)\\
&F = f^2\\
&f\sigma_e = q - \sigma_e\\
&\sigma_e = \sigma_y + h\overline{\epsilon}^p\\
&\overline{\epsilon}^{p2} = \frac{2}{3}\epsilon^p : \epsilon^p
\end{aligned} \tag{5}$$

where σ^d and ϵ^p indicate respectively the stress deviatoric tensor and the plastic strain tensor, D is the elastic material stiffness tensor, μ is the shear modulus, λ, n, q, σ_e, $\overline{\epsilon}^p$ and f are the plastic multiplier, the normal to the surface charge, the equivalent stress, the effective stress, the equivalent plastic strain and the load function respectively, σ_y and h represent respectively the yield stress and the module of work hardening for an elastoplastic material with linear work hardening, the regularization functions H and G are introduced into the model to overcome the difficulty of singularities (see Appendix A), η_1, η_2 and η_4 are the regularization parameters and $\dot\epsilon_c$ is a characteristic strain rate, Z, Den and F are auxiliary variables introduced with the aim to write all relationships in a quadratic form, the sign $(\dot\cdot)$ denotes the derivative with respect to the time parameter t.

In order to evaluate the ability of the proposed algorithm in a typical process (elastic, plastic, elastic unloading), we impose a time-dependent load parameter $C(t)$ given by the following hyperbolic relation:

$$\left(C(t) - \frac{C_m}{T_m}t\right)\left(C(t) - \frac{C_m}{T_m}(2T_m - t)\right) = \eta_3 C_m^2 \tag{6}$$

where C_m and T_m are the given parameters and η_3 is a regularization parameter. The problem defined by Eqs. (4)–(6) can be written in the form (1) if we introduce the following mixed variable:

$$U = {}^t\langle u, \sigma, \epsilon^p, n, q, \sigma_e, f, \lambda, G, F, \overline{\epsilon}^p, Den, \xi, H\rangle \tag{7}$$

In the following section, we discuss how to establish a continuation procedure based on the ANM to solve the structural problem (4)–(6) with an elastic–plastic constitutive law.