Finite Elements in Analysis and Design 101 (2015) 34–45

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

A new adaptive multiscale method based on the estimate of residual forces for static analysis of heterogeneous materials

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ARTICLE INFO

Article history:
Received 15 September 2014
Received in revised form
3 April 2015
Accepted 3 April 2015
Available online 28 April 2015

Keywords:
Heterogeneous materials
Adaptive multiscale method
Multi-node extended multiscale finite element method (multi-node EMsFEM)
Residual force
Multiscale computation

ABSTRACT

A new adaptive multiscale method (AMM) is developed based on the estimate of residual forces for static analysis of heterogeneous materials. The AMM is established by combining multi-node extended multiscale finite element method (multi-node EMsFEM) with a new proposed macroscopic node adaptive algorithm. In our previous multiscale computations, macroscopic nodes are placed uniformly along each edge of multi-node coarse element without considering local strain or displacement gradient. In this paper, to optimize the distribution of macroscopic nodes, a new adaptive algorithm is proposed based on the estimate of residual forces. Numerical experiments have indicated that residual forces exist even for linear elastic problems. For boundary external loading cases, residual forces only exist on the edges of coarse element. Besides, computations indicate that residual forces can reflect local relative errors in the multi-node EMsFEM computations. Thus it is reasonable and suitable to take residual forces as local relative error indicators in the multi-node EMsFEM computations. Finally, the AMM is developed based on this idea. To verify the validity of this proposed method, three typical numerical examples are carried out. The examples demonstrate that nearly optimal distributions of macroscopic nodes can be obtained by employing the proposed AMM.

1. Introduction

As is well known, composites are more and more widely applied in the aeronautics, astronautics, automotive and other industrial areas due to their excellent performances. Almost all composites have multiscale features [1,2]. The schematic diagram of heterogeneous materials is shown in Fig. 1, in which the minimum material size ε is usually much smaller than the unit cell size h. Therefore, the grids must be refined to make their size smaller than the minimum material size when calculating the mechanical behaviors of the composite structures with complex microstructures by using direct methods, such as the finite element method (FEM) and the finite difference method (FDM). It will cost lots of computational resources. In this context, many multiscale computation methods which can not only save computational resources but also ensure computational accuracy have been developed in recent decades, such as the computational homogeneous method [3–7], the representative volume element method [8–12], the heterogeneous multiscale method [13–15], the multiscale quasicontinuum (QC) method [16–20], the multiscale finite element method (MsFEM) [21,22] and its branches [23,24].

To capture local properties of heterogeneous materials more accurately and reasonably, many adaptive algorithms have been presented to solve the multiscale problems with localized effects. Abdulle and Nonnenmacher [25] presented a posteriori error analysis for the elliptic homogenization problems discretized by the finite element heterogeneous multiscale method. Jenny et al. [26] developed an adaptive multiscale finite-volume (MSFV) method for the multiphase flow and transport in heterogeneous porous media. Larson [2] developed a new adaptive multiscale finite element method using the variational multiscale framework together with a systematic technique for approximation of the fine scale part of the solution. Zheng et al. [27] studied the stationary incompressible Navier–Stokes equations using the variational multiscale (VMS) method with h-adaptive technique. Ghosh et al. [28] developed an adaptive concurrent multi-level computational model for multi-scale damage analysis of composite structures due to the de-bonding at the fiber–matrix interface. Vernerey and Kabiri [29] employed and extended this model for the elasticity problems with periodic microstructures. More recently, Vernerey and Kabiri also performed this model for fracture and crack propagation in heterogeneous media [30]. Temizer and Wriggers [31] performed a finite deformation analysis of microscopic heterogeneous structures using an adaptive multiscale resolution strategy based on the homogenization formulations. Lee et al. [32] described a sequential implicit multiscale

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http://dx.doi.org/10.1016/j.finel.2015.04.001
0168-874X/

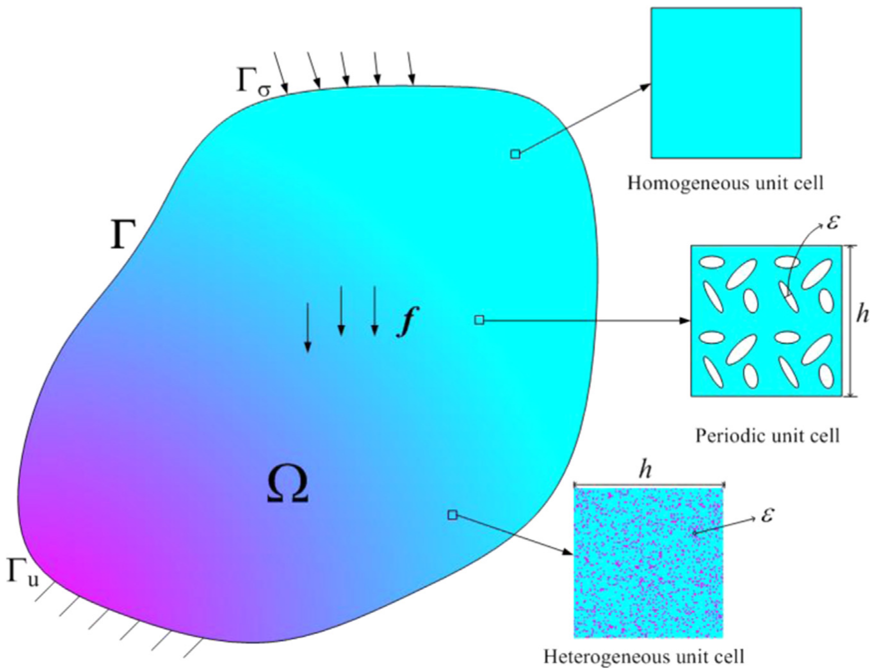

Fig. 1. Schematic diagram of heterogeneous materials.

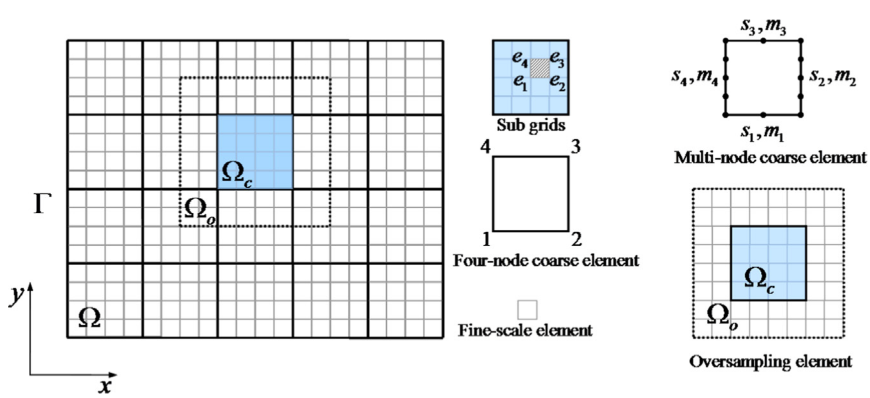

Fig. 2. Schematic diagram of the multi-node EMsFEM.

finite-volume framework for coupled flow and transport with general prolongation and restriction operations for both pressure and saturation where three adaptive prolongation operators for the saturation are used. He and Ren [33] presented an adaptive multiscale finite element method for solving the unsaturated water flow problems in heterogeneous porous media with many scales. Hajibeygi and Jenny [35] introduced a space-time adaptive iterative multiscale finite volume method for the multiphase flow problems.

To investigate the static and dynamic problems of heterogeneous materials uniformly, Zhang and Liu [36,37] developed a multiscale method to solve multiscale problems in computational solid mechanics. A multi-node coarse element was presented by them to simulate the mechanical behaviors of heterogeneous materials with complex deformation. Besides, by virtue of this multi-node coarse element, Liu [34] presented a *p*-adaptive multiscale algorithm, established based on the relative error estimate of strain energy density of the coarse element between two adjacent iterative steps, to find out a nearly optimal distribution of macroscopic nodes on the fixed coarse-scale meshes.

In this paper, we develop a new adaptive multiscale method based on the estimate of residual forces for static analysis of heterogeneous materials. This adaptive algorithm can be implemented easily by comparison with the aforementioned ones. It is promising to popularize this method in computational engineering fields. By using this new adaptive method, the macroscopic nodes can be distributed reasonably and automatically according to the practice needs. This is to say that: more nodes will be placed on the domain with larger displacement gradient, and fewer will be distributed at the region with smaller displacement gradient.

This paper is organized as follows. In the next section, the multi-node EMsFEM is reviewed briefly and the construction processes of numerical base functions of multi-node coarse element are introduced. In Section 3, the residual forces in the multi-node EMFEM computations for static analysis of heterogeneous materials are investigated in detail. In Section 4, the macroscopic nodes adaptive algorithm is presented based on the estimate of residual force. In addition, to verify the reasonableness and effectiveness of the presented algorithm, some typical numerical