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A numerical model for the scattering of elastic waves from a non-axisymmetric defect in a pipe

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ABSTRACT

Ultrasonic guided waves are used in the non-destructive testing of pipelines. This involves launching an elastic wave along the wall of the pipe and then capturing the returning wave scattered by a defect. Numerical study of wave scattering is often computationally expensive because the shortest wavelength is often very small compared to the size of the pipe in the ultrasonic frequency range. Furthermore, the number of the scattered wave modes from a non-axisymmetric defect in the pipe can be large and separation of these modes is difficult in a conventional finite element method. Accordingly, this article presents a model suitable for studying elastic wave propagation in waveguides with an arbitrary cross-section in the time and frequency domain. A weighted residual formulation is used to deliver an efficient hybrid numerical formulation, which is applied to a long pipeline containing a defect of arbitrary shape. The problem is solved first in the frequency domain and then extended to the time domain using an inverse Fourier transform. To separate the scattered wave modes in the time domain, a technique is proposed whereby measurement locations are arranged axially along the pipe and a two dimensional Fourier transform is used to present data in the wavenumber–frequency domain. This enables the separation of highly dispersive modes and the recovery of modal amplitudes. This has the potential to reveal more information about the characteristics of a defect and so may help in distinguishing between different type of defects, such a cracks or regions of corrosion, typically found in pipelines.

1. Introduction

Guided ultrasonic waves are used in the non-destructive testing of pipelines. The guided waves normally take the form of a pulse with a narrow frequency bandwidth, which is launched along the pipe wall and is then scattered when it hits a defect in the pipe, such as a crack or region of corrosion. Following scattering by the defect the returning wave is detected and interrogated and the aim of the method is to infer the presence of the defect and if possible the geometry of the defect. The scattering of guided ultrasonic waves from a general defect is a three-dimensional problem and so this presents a significant computational challenge that potentially requires a large number of degrees of freedom. Consequently computation time can quickly become prohibitive for relatively long pipe lengths. To overcome this problem, this article applies a hybrid numerical approach in order to deliver an efficient three dimensional methodology for the propagation of elastic waves in a pipe. It is demonstrated that this type of approach is sufficiently efficient to permit the generation of predictions for scattering from arbitrary defects located in long lengths of pipe in both the frequency and time domain.

The development of three dimensional numerical models suitable for modelling large structures including pipelines continues to present a significant challenge. Current strategies include attempting to take advantage of symmetries present in a structure, or using finite element discretisations that are localised around a defect. For example, one may take advantage of symmetry in a pipeline and reduce the problem to two dimensions. Heidary and Ozevin recently accomplished this for an axisymmetric pipe under non-axisymmetric loading conditions [1], although this method cannot readily be applied to wave scattering problems from a general defect in a pipe because the loading function for a general defect remains unknown. Moreover, one must still mesh the entire pipe length which reduces the effectiveness of the method for long pipes. Alternatively, one can treat the pipe wall as a one dimensional structure and assume that the displacements and associated stresses and strains are constant over the pipe wall [2,3]. This will give a good approximation of the true behaviour for those modes in which the through thickness displacement profile is close to being constant, although this will be less successful for those modes for which this is not the case. Therefore, as the frequency of excitation is increased this approach

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is unlikely to capture accurately the behaviour of all modes propagating within a pipe.

To overcome the limitations of two dimensional models one must return to a three dimensional approach that delivers a full numerical discretisation of the defect as well as the surrounding structure. For example, Casadei et al. [4] presented a multi-scale finite element approach to study localized defects in plates. A multi-scale approach delivers a significant reduction in elements placed well away from a defect and so improves the efficiency of the model. However, this method still demands that the entire structure is meshed and so for large structures such as plates recent efforts centre on discretising only the immediate vicinity surrounding a scattering object. For example, Velichko and Wilcox [5] use finite elements to discretise the region surrounding a scatterer and then asymptotic Green's functions to reconstruct the solution outside this region. This has the advantage of lowering the number of degrees of freedom required to analyse a relatively small scattering object located in a much larger structure; however, in order to suppress reflections from the boundaries in the outer region it is necessary to include an artificial absorbing region. Velichko and Wilcox demonstrate that this method can be applied to guided wave scattering from a circular hole in a plate, and this approach was later applied to irregular defects by Moreau et al. [6]. The addition of an absorbing layer, which is often called a perfectly matched layer, has also been widely applied by other authors and is popular in commercial finite element software. For example, Żak et al. proposed an absorbing layer with a particular damping profile to suppress reflections in a finite element model for elastic wave propagation in unbounded structures [7]. It is of course possible to apply this type of approach to the study of guided waves in pipelines; however, the absorbing boundary or perfectly matched layer demands extra degrees of freedom and this leads to computational inefficiencies. Moreover, the absorbing region does not fully absorb the outgoing waves and so some reflection is inevitable, especially in three dimensional applications where high order propagating modes are difficult to attenuate. Accordingly, where possible it is desirable to seek methods that avoid the use of absorbing layers.

In the study of guided waves it is common to encounter long uniform sections of waveguide surrounding a relatively small scattering object or defect. It is desirable to take advantage of regions of uniformity and this can be accomplished in an efficient way by using a normal mode expansion. For example, Cho and Rose [8,9], and Zhao and Rose [10] proposed a hybrid boundary element method for analysing the scattering of Lamb and shear horizontal waves in a plate. Here the elastodynamic boundary integral equation is mapped onto an analytical normal mode expansion for the uniform section of the plate and this enabled the study of mode conversion by arbitrary defects. This method demonstrates a more efficient approach to studying guided waves as it does not rely on adding artificial absorbing regions. However, the method relies on the use of analytic expression for the propagating eigenmodes and so it is desirable to look at ways of removing this restriction so one can study waveguides with irregular cross-sectional geometries, as well as those applications where obtaining analytic solutions is challenging. Accordingly, it is sensible to extend this type of approach to include a numerical solution of the governing eigenequation for the uniform section of the waveguide. To this end a hybrid normal mode/finite element methodology has long been used in the study of elastic waves, although its application to pipes is still relatively limited. For example, Datta and Shah [11] applied the method to the scattering of shear waves in a plate, and Baronian et al. [12] examined the more general case of scattering from an arbitrary defect, although this was limited to a two dimensional waveguide. For pipes, Zhuang et al. [13] applied a hybrid formulation in the study of scattering from cracks in welds, although their analysis was restricted to an axisymmetric problem in order to permit the use of a Rayleigh Ritz approach for solving the eigenproblem on either side of the defect. Bai et al. [14] extended the work of Zhuang et al. [13] to non-axisymmetric circumferential cracks by combining analytic solutions for the pipe eigenmodes and coupling these to a numerical discretisation for the crack, although this was limited to an infinitely thin crack and, like Cho and Rose [8,9], they relied upon analytic eigensolutions. However, by subdividing the scattering from the crack into a symmetric and antisymmetric problem, Bai et al. were able to show that one may generate reflection coefficients for a non-axisymmetric problem in a pipe.

An alternative method for computing the eigenmodes in a uniform section of the pipe was proposed by Zhou et al. [15], who used the wave finite element (WFE) method to solve the governing eigenproblem. This numerical solution for the eigenmodes enabled Zhou et al. to link their model to a finite element discretisation of the defect in a computationally efficient way. Zhou et al. used a hybrid WFE and finite element scheme to study axisymmetric and non-axisymmetric defects in pipes and presented predictions for reflection coefficient in the frequency domain. The majority of the results presented by Zhou et al. are for a two dimensional formulation. Furthermore, it is known that some numerical issues are present with the WFE method [16,17] and so it appears appropriate to investigate alternative methods for three dimensional problems. One possible approach is to find the eigenmodes for a waveguide by directly solving the governing eigenequation. This method is often referred to in the elastodynamic literature as the semi analytic finite element (SAFE) method; however it has also been used in the acoustic waveguide literature where there is no such terminology [18,19]. A hybrid SAFE-FE method was recently applied to elastic wave propagation in a solid cylinder by Benmeddour et al. [20]. This method uses a full three dimensional discretisation of a small region surrounding the non-axisymmetric crack and then uses the two dimensional finite element mesh on the surface of this region to directly solve the governing eigeneqaution. Accordingly, this method removes the need for absorbing boundaries remote from the defect and/or separating three dimensional slices of the pipe for solving the eigenproblem. This makes the SAFE-FE hybrid method very efficient and so it has also been used, for example, to study axisymmetric defects in coated pipes [21,22]. The purpose of this article is the application of the SAFE-FE method to pipe in a way that permits the solution and separation of all propagating modes in the time domain.

The hybrid methods developed so far tend to be limited to the frequency domain, at least when studying three dimensional problems. This article will demonstrate that it is possible to generate predictions in the time domain for an arbitrary non-axisymmetric defect in a waveguide of arbitrary cross-section. The work presented here adopts a hybrid finite element method similar to that proposed by Benmeddour et al. [20], although a different formulation is used here. Benmeddour et al. use the principle of virtual work and apply a variational formulation to derive the governing SAFE-FE equations, whereas in this article we will use the alternative weighted residual method (WRM). The WRM is in principle a more general approach than the variational method and so this method is presented here for elastic wave propagation in pipes. Galerkin's method is then used to solve the problem and this is of course equivalent to a variational approach, but it will be shown that by using the WRM method one may arrive at a final governing equation that avoids the multiplication of global matrices seen in the variational approach of Benmeddour et al. [20]. The avoidance of these additional global matrix multiplications is potentially advantageous when one is required to make many repetitive solutions of the problem, as will be the case when Fourier transforms are used here to generate time domain predictions. Accordingly, this article begins by presenting a hybrid SAFE-FE formulation in Section 2 using a weighted residual approach. The frequency domain methodology is presented first and time domain simulations are then obtained following an inverse Fourier transform