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Fuzzy variational principle for modal analysis of structures and its application

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ABSTRACT

In this paper, the modal analysis to estimate eigenvalue and eigenvector of structures with fuzzy parameters is presented. By introducing fuzzy parameters into the Rayleigh quotient variation, the fuzzy variational principle is developed. The fuzzy finite element method is proposed as the application of the fuzzy variational principle. The proposed fuzzy finite element method can obtain fuzzy eigenvalues and fuzzy eigenvectors directly. However, when the conventional fuzzy finite element methods are used for fuzzy analysis, the fuzzy parameters have to be transformed into the interval parameters before the calculation and the fuzzy results are subsequently constructed by the interval results. The proposed method avoids these roundabout procedures in comparison with the conventional fuzzy finite element methods. Therefore, the proposed method can reduce the computational cost and increase computational efficiency. Numerical examples are presented to illustrate the validity and feasibility of the proposed method.

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1. Introduction

The modal analysis of structures plays an important role in the design and analysis of structural systems. However, real engineering structures are usually complicated systems, which possess various uncertainties due to the manufacture errors, measurement errors and other factors. Consequently, eigenvalues and eigenvectors of structural systems are also uncertain.

The modal analysis of structures with uncertain parameters has been the subject of some scientists and engineers for many years. Deif [1] proposed a method to obtain interval eigenvalues for the standard interval eigenvalue problem. Considering the characteristics of structural systems through the non-negative decomposition of the mass and stiffness matrices, Qiu et al. [2] presented an approach for solving the generalized interval eigenvalue problem. Sim et al. [3] developed a modal analysis method to estimate modal parameters, frequency response function and mode shapes of structures with uncertain-but-bounded parameters via interval analysis. Zhu and Wu [4] proposed a stochastic finite element method for both distinct and repeated eigenvalues and derived formulae for the variance and covariance of the eigenvalues and eigenvectors. Based on the perturbation approach, Song et al. [5] applied a method to obtain the stochastic sensitivity of eigenvalues and eigenvectors, which does not greatly increase the computational cost compared to deterministic analysis. Chen and Rao [6] developed a methodology using the fuzzy finite element method for eigenvalues and eigenvectors of imprecisely defined systems. Based on α-cut discretization of fuzzy numbers and Taylor's expansion, Massa et al. [7] presented an efficient methodology to calculate fuzzy eigenvalues and eigenvectors of finite element structures defined by fuzzy parameters. As we can see above, there are several methods to deal with uncertain analysis of structures: the interval-based methods; the stochastic-based methods; and the fuzzy-based methods, etc. When the system parameters are described in linguistic or imprecise terms, the fuzzy-based methods are appropriate [8].

Among numerical methods for the fuzzy analysis of structural systems, the fuzzy finite element method is a widely used method. Based on the theory of fuzzy sets, Valliappan and Pham [9] proposed a finite element model to take account of the uncertainty in the soil behavior and considered the elastic modulus and Poisson's ratio as fuzzy numbers. Rao and Sawyer [8] developed a fuzzy finite element approach for the static analysis of structural systems with parameters described in linguistic or imprecise terms, this method can be applied to analyze problems involving vaguely defined geometry, material properties, external loads and boundary conditions. Wasfy and Noor [10] presented a fuzzy finite element method for predicting the dynamic response and evaluating the sensitivity coefficients of flexible multibody systems with fuzzy parameters. By using the interval finite element method, Moens and Vandepitte [11] and Gersem et al. [12] introduced a numerical algorithm to calculate frequency-response functions of damped finite element models with fuzzy uncertain parameters. Based on a sensitivity analysis and a high

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order Taylor series expansion, Massa et al. [13] proposed a fuzzy procedure for the static design of imprecise structures. This methodology can deal with structures modeled by imprecise material or geometric parameters and subjected to imprecise boundary conditions and imprecise external loads. Verhaeghe et al. [14] discussed the application of the concept of interval fields and developed a fuzzy finite element analysis technique for structural static analysis based on interval fields to cope with the problem that mutual dependency between multiple uncertain model parameters can't be included in the analysis. Farkas et al. [15] applied a reanalysis-based finite element method for static structural analysis and this approach is explicitly suited for optimisation-based black-box techniques. Nearly all these fuzzy finite element methods are based on the interval calculation. When these fuzzy finite element methods are used for the fuzzy analysis, the fuzzy parameters have to be transformed into interval parameters through the concept of α-level to calculate the interval results at each α-level of interest. The fuzzy results are subsequently constructed by all the obtained interval results. So the computational cost of these methods is expensive.

The variational principle is one of the important theoretical bases of the finite element method [16]. Many scientists studied the deterministic variational principle and there are a lot of research works on deterministic variational principles [16–18]. In the field of uncertain variational principle, scientists focused their attentions on stochastic variational principle. By combining the Hu – Washizu variational principle with second-order perturbation techniques, Liu et al. [19] established a probabilistic Hu – Washizu variational principle for the probabilistic finite element method. Hien and Kleiber [20] formulated a stochastic Hamilton variational principle for dynamic problems of linear continuum. The stochastic Hamilton variational principle allows incorporation of probabilistic distributions into the finite element analysis. Elishakoff et al. [21] proposed a stochastic variational principle for the mean and covariance functions of the displacement of stochastic beams and based on the stochastic variational principle, Galerkin and Rayleigh – Ritz methods were presented to find probabilistic characteristics of the response. Impollonia and Elishakoff [22] extended the stochastic variational principle to the probabilistic response of shear beams with stochastic flexibility, and subjected to deterministic static loads. Expanding all the random quantities involved in the energy functional through second-order perturbation techniques, Yang et al. [23] presented the stochastic transient variational principle for vibration analysis of linear continuum. Based on the stochastic variational principle and second-order perturbation techniques, Yang et al. [24] developed a stochastic spline Ritz method to analyze the structures consisting of elastic beams and plates by a local average of the homogeneous random field. However, there are no research works on the fuzzy variational principle until Yang et al. [25] proposed the fuzzy variational principle for static analysis of structural systems with fuzzy parameters.

In this paper, based on Rayleigh quotient variation and perturbation techniques, the fuzzy variational principle for modal analysis of structural systems is presented. The fuzzy finite element method is developed as the application of the fuzzy variational principle. The proposed fuzzy finite element method can obtain the fuzzy result directly. Compared with the conventional fuzzy finite element methods, the proposed method can reduce the computational cost. Numerical examples are presented to illustrate the practicability of the proposed method.

2. Fuzzy variational principle

For the sake of simplicity, in this section, indicial notation is employed with indices repeated twice implying summation.

The Rayleigh quotient variation is one of the important theoretical bases of the finite element method for modal analysis of structural systems. Its general form can be described by

$$\omega^2 = st\frac{V_{\max}}{T}, \tag{1}$$

where ω is the natural frequency, $V_{\max}$ is the maximum of the potential energy of elastic, T is the kinetic energy coefficient, and st denotes the operation to find the stationary value of a functional.

If there are fuzzy factors in structural systems, the expression of the Rayleigh quotient variation will be a fuzzy functional, which can be written as

$$\tilde{\omega}^2 = st\frac{\tilde{V}_{\max}}{\tilde{T}}, \tag{2}$$

where $\tilde{\omega}$ is the fuzzy natural frequency, $\tilde{V}_{\max}$ is the fuzzy maximum of the potential energy of elastic, $\tilde{T}$ is the fuzzy kinetic energy coefficient.

$\tilde{V}_{\max}$ can be expressed as

$$\tilde{V}_{\max} = \frac{1}{2}\int_V \tilde{D}_{ijkl}\tilde{\varepsilon}_{ij}\tilde{\varepsilon}_{kl}dV, \tag{3}$$

where $\tilde{D}_{ijkl}$ is the fuzzy tensor of elastic moduli, $\tilde{\varepsilon}_{ij}$ is the fuzzy strain tensor.

$\tilde{T}$ can be expressed as

$$\tilde{T} = \frac{1}{2}\int_V \tilde{\rho}\tilde{u}_i\tilde{u}_i dV, \tag{4}$$

where $\tilde{\rho}$ is the fuzzy density, $\tilde{u}_i$ is the fuzzy displacement vector.

The constitutive relation is

$$\tilde{\sigma}_{ij} = \tilde{D}_{ijkl}\tilde{\varepsilon}_{kl}, \tag{5}$$

where $\tilde{\sigma}_{ij}$ is the fuzzy stress tensor.

For the sake of simplicity, we can build the following equations

$$\tilde{U} = \frac{1}{2}\tilde{D}_{ijkl}\tilde{\varepsilon}_{ij}\tilde{\varepsilon}_{kl}, \tag{6}$$

and

$$\tilde{M} = \frac{1}{2}\tilde{\rho}\tilde{u}_i\tilde{u}_i. \tag{7}$$

Then Eq. (2) can be written as

$$\delta\int_V \tilde{U}dV = \tilde{\lambda}\delta\int_V \tilde{M}dV, \tag{8}$$

where $\tilde{\lambda} = \tilde{\omega}^2$ is the fuzzy eigenvalue.

All the fuzzy variables of structural systems including geometric parameters, physical parameters, external forces, and boundary conditions, etc., can be denoted with the fuzzy vector field

$$\tilde{\boldsymbol{X}} = \left\{\tilde{X}_1, \tilde{X}_2, \cdots, \tilde{X}_r\right\}^T, \tag{9}$$

where r is the number of fuzzy parameters. The fuzzy field $\tilde{\boldsymbol{X}}$ can be decomposed as follows

$$\tilde{\boldsymbol{X}} = \boldsymbol{X}_0 + \tilde{\boldsymbol{\beta}}, \tag{10}$$

where

$$\boldsymbol{X}_0 = \{X_{01}, X_{02}, \cdots, X_{0r}\}^T, \quad \tilde{\boldsymbol{\beta}} = \left\{\tilde{\beta}_1, \tilde{\beta}_2, \cdots, \tilde{\beta}_r\right\}^T, \tag{11}$$

$X_{0m}(m = 1, 2, \ldots, r)$ is the real value of $\tilde{X}_m(m = 1, 2, \ldots, r)$, and $\tilde{\beta}_m(m = 1, 2, \ldots, r)$ is the fuzzy perturbation quantity with zero real value.

$\tilde{\lambda}$ is dependent on the fuzzy vector field $\tilde{\boldsymbol{X}}$, so we can expand $\tilde{\lambda}$ about the real value $\boldsymbol{X}_0$ of the fuzzy field $\tilde{\boldsymbol{X}}$ via Taylor series, and retain up to first-order terms only

$$\tilde{\lambda} = \lambda_0 + \tilde{\beta}_m\lambda_{,m}, \tag{12}$$

where λ_0 denotes λ evaluated at the real value of the fuzzy field, $\lambda_{,m}$ denotes the first-order partial derivative of λ to the field variable

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