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Parallel, second-order and consistent remeshing transfer operators for evolving meshes with superconvergence property on surface and volume

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ABSTRACT

This paper investigates several field transfer techniques that can be used to remap data between three-dimensional unstructured meshes, either after full remeshing of the computational domain or after mesh regularization resulting from an ALE (Arbitrary Lagrangian or Eulerian) formulation. The transfer is focused on state (or secondary) variables that are piecewise discontinuous and consequently only defined at integration points. The proposed methods are derived from recovery techniques that have initially been developed by Zienkiwicz et al. in the frame of error estimation. Obtaining a higher order interpolation with the recovered fields allows reducing the inescapable diffusion error resulting from the projection on the new mesh. Several variants of the method are investigated: (a) either based on nodal patches or on element patches, (b) by enforcing the balance equation in a weak sense or in a strong sense or not, (c) by using first or second interpolation orders. A special attention is paid to the accuracy of the transfer operators for surface values, which can play a first order role in several mechanical problems. In order to take into account the constraint due to parallel calculations, a new iterative approach is proposed. All methods are evaluated and compared on analytical tests functions, both for the ALE formulation and for full remeshings, before being applied to an actual metal forming problem. In all studied examples, in addition to improved accuracy, higher order convergence rates are observed both for volume and surface values, so providing quite accurate transfer operators for various applications.

1. Introduction

For a large class of nonlinear problems, such as forming processes or highly heterogeneous loadings, the optimal mesh configuration continually changes throughout the deformation process, requiring successive mesh adaptations during the numerical simulation. Therefore, the problem resolution requires the transfer of data fields such as pressure, temperature, velocity, stress and strain between different meshes, and the quality of the numerical simulation highly depends on the quality of transfer operator allowing continuing the computation on a new mesh. In Arbitrary Lagrangian or Eulerian (ALE) formulations, such data transfer takes place at each time step [1]. In Lagrangian formulations, it is activated at each remeshing step. Several important aspects of the transfer operator, such as consistency with constitutive equations, conservation of equilibrium equations or bound on extrema need to be considered to have a stable and reliable data transfer [2]. However, the transferring approach highly depends on the type of variables which needs to be transferred. Most of physical data encountered in engineering applications can be roughly categorized into variables which are either continuous or discontinuous throughout the domain. The continuous variables (such as displacements, velocity and temperature) are often stored at nodes; they will be referred to as *P1 variables* in the present paper where a linear finite element interpolation is used. The discontinuous variables (such as stress and strain tensors) are stored at the Gauss points of the element; they will be referred to as *P0 variables* in the present paper where a single Gauss point is used at the center of the element. The techniques that are developed to transfer *P1 variables* are hardly applicable to *P0 variables* because of their discontinuous characteristic. Transfer of P0 variables consequently requires more sophisticated techniques and is the main subject of this paper.

In literature, the oldest method used for P0 transfer is a direct P0 operator. The value at an integration point of the new mesh is directly copied from the nearest integration point of the old mesh. Srikanth et al. [3] for instance further improved this direct approach

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by introducing weighted averages. If the nearest integration point of the old mesh is close enough to the considered integration point, then the variable is approximated by its value, otherwise a weighted average over the values of the integration points located at a selected distance of the considered point is performed. The second class of method is based on the weak conservation of the transferred field in the domain. Shashkov et al. [4,5] formulated a local-bound-preserving conservative transfer operator with 1st and 2nd order of accuracy. An approximated integration of the constructed function on the new grid is used to obtain the mean value in the cell. Liska et al. [6] extended the technique by an idea derived from flux corrected transfer [7] and formulated flux corrected transfer by introducing a new local measure of accuracy which is derived from L_1 norm of the error. Lin et al. [8] proposed a scheme based on the concept that the error caused by transfer can be confined within particular areas of the mesh that have been significantly changed. Inside these "worse areas", they tested four methods including linear interpolation, particle transfer, 1st order integral transfer and high order essentially non-oscillatory transfer, before concluding that only worse area is needed to be transferred.

Following a similar approach, Rachid [9] proposes to associate a "tributary region" to any integration point of the mesh. It allows writing a piecewise discontinuous interpolations over the domain, both for the old and new meshes. The minimization of the L^2 norm of the difference between these two discretizations integrated over the entire domain directly provides the integration point values of the new mesh. The main difficulty consists in computing the integration terms of the L^2 norm with non-matching meshes. In [10], in the frame of the finite volume method, the integration of the difference norms is based on using the mortar elements technique; it allows developing a very conservative operator.

The general idea of this paper is that, whichever the utilized transfer operator and conservative as it can be, transfer always results into a loss of information due to diffusion during the change of mapping. It is then believed that it is possible to reduce and control this loss of information by enriching the transferred field beforehand, in such a way that this enrichment can compensate for the inevitable loss. This enrichment is derived from techniques which have initially been developed for error estimation based on the comparison between the finite element field and a recovered enhanced field, as originally proposed by Zienkiewicz and Zhu [11]. In this particular context, it has been shown the global L^2 norm does not allow building a sufficiently accurate recovered field (because it smooths the solution too much) and that local approximations should be built using superconvergence properties [12,13]. This paper follows this main idea to build recovered fields of a higher order than the initial field.

At first, we introduce the interpolation method. It has often been used in literature following two approaches: (a) direct interpolation [6,14] and (b) nodal recovery based techniques [2,15,16]. The direct interpolation approach relies on using a set of field values at Gauss points (of the old mesh) located in the vicinity of the considered Gauss point (of the new mesh) for building a local and continuous approximation. In the work of Liszka et al. [6,14] with irregular meshes, the interpolation method was based on a Taylor series expansion of unknown functions combined with the minimization of interpolation error. A shortcoming of this approach is the lack of continuity between the set of considered Gauss points. Villon et al. [17,18] developed a transfer operator based on the diffuse approximation, also referred to as the Moving Least Square method, which is both local and continuous over the entire domain. The reconstruction of the stress field is carried out using operators that intrinsically preserve the local equilibrium (balance equation and stress admissibility). On the other hand, in the second approach of nodal recovery techniques [2,15,16,19], the P0 variables of the old mesh are first projected onto nodal points to build a continuous interpolation. The nodal points values on new mesh are then computed by simple interpolation of old mesh nodal values using the shape-functions. The P0 variables at the Gauss points of the new mesh are finally obtained by employing the shape functions of new mesh. Peric et al. [2] formulated a nodal recovery based transfer operator and discussed the different aspect related to equilibrium conditions, consistency with the constitutive equation, geometric issues and diffusion of state variables between successive meshes. Khoei et al. [20] pointed out that by transferring the information to the nodes and recalculating them at new Gauss points, the equilibrium of the system is violated, even if the mesh is not changed. Dureisseix et al. [16] also formulated a nodal recovery based transfer operator by using an approximated integral of the Jacobian transformation of the point needed to be transferred with weights of those points to recover the nodal field.

Mediavilla et al. [19] modeled a quasi-static ductile crack propagation by using a nodal recovery based transfer operator. They used a standard extrapolation and nodal averaging technique to recover the state variable from the discrete values known at the integration points, although such technique will produce numerical diffusion. Hinton et al. [21] used a global least squares approximation and Loubignac et al. [22] used an iterative procedure to construct a smooth stress field to reduce numerical diffusion. Less diffusion can be achieved by computing the new integration point variables directly from the continuous stress field on the old mesh [15]. Yet, this introduces more inconsistencies as was shown in Pavanachand [23]. The consistency issue of the transfer operator is well explained by Camacho et al. [24], clearly stating that consistency among field variables may be lost due to the fact that nonlinear relations between them are not carried over correctly by a linear transfer operator.

In 1992, in the frame of error estimation, Zienkiewicz et al. [12,13] introduced the concept of Superconvergent Patch Recovery (SPR), where nodal field is evaluated by determining a polynomial expansion over a patch of elements sharing the node. Several modified and alternative recovery technique based on SPR have been proposed over the years to increase its robustness and accuracy. Boroomand et al. [25] proposed Recovery by Equilibrium in Patches (REP) which avoids the specification of superconvergent points. Gu et al. [26] modified SPR by using integration points as sampling points and introducing additional nodes, which increased the performance of SPR for nonlinear problems remarkably. Wiberg et al. [27,28] presented an improvement to SPR by extending the technique to use element patches rather than nodal patches, and extended the SPR to SPRE (penalizing the violation of the equilibrium) and SPREB (by including a least square fit of the boundary conditions). SPR technique has been more often used for the purpose of FEM error estimation rather than remapping. Khoei et al. [20] formulated a 3D data transfer operator based on SPR model for a plasticity problem by implementing 0 (C_o continuity), 1st (C_1 continuity) and 2nd (C_2 continuity) order polynomial while formulating SPR, clearly stating the effect of each variant. When dealing with boundary variables, Khoei used higher order patch to maintain the same order of accuracy on the boundary as for the internal points.

All these techniques apply to continuous fields and implicitly assume that the finite element method is converging, in other words, that a higher order interpolation of the field to be transferred provides a higher order solution. This is the case of problems including discontinuities such as bi-materials.

In the present paper, eight transfer operators based on different interpolation approaches are formulated and tested against each other over benchmark problems. Parallel aspects of transfer operator are discussed. A new modified iterative SPR recovery based transfer operator is proposed, which maintains its order of