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# Post-buckling analysis of composite plates under combined compression and shear loading using finite strip method

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ABSTRACT

The post-buckling analysis of laminated plates under combined shear and compression is presented using the nonlinear finite strip method. Similar to the end shortening strain for compression, the skewed angle strain is uniquely proposed for in-plane shear action. The nonlinear governing equations under the skewed angle and end shortening are solved numerically using the Newton–Raphson method. The numerical finite element analysis is conducted to validate the proposed method, and a parametric study is performed to show the post-buckling behavior of composite plates. It is noted that the resulting average longitudinal and transverse section forces induced by the out-of-plane deflection or so called the non-linear strains cannot be ignored when compared to the average shear section force for the case of pure shear action. Also, when the out-of-plane deflection becomes large enough under combined compression and shear action, the average compression section force will transit to the average tensile section force in the longitudinal direction. The present analysis is capable of simulating the post-buckling behavior under the combined shear and compression action.



## 1. Introduction

Composite materials have been widely used in civil aircrafts and aerospace vehicles because of their high specific strength and stiffness. The rectangular flat plate supported at its edges is one of the basic elements in composite structures. The structure is often subjected to a combination of in-plane loads, and it is noteworthy that most of the investigations appeared in the literature dealt with the post-buckling analysis of structures loaded only in in-plane compression. There are relatively few investigations about post-buckling of structures loaded in pure shear or combination of shear and compression. Stein [1,2] presented a Levy-type solution for the post-buckling analysis of long plates loaded in combined shear and compression. The ordinary nonlinear differential equations were derived to replace the nonlinear partial differential equations in plate theory. The method of analysis was based on the principle of virtual work in conjunction with the assumption that the deformations in the longitudinal direction are represented by a few important terms of a Fourier series. Zhang and Matthews [3,4] presented a nonlinear analysis of flat and curved panels of laminated composite materials under either compression or in-plane shear loading. In the analysis, a pair of governing equations in the von Karman sense is solved in conjunction with simply supported boundaries, and the governing equations were obtained using the virtual displacement and virtual force principles. They further studied the effect of shear direction on the post-buckling behavior of symmetric laminated composite plates and revealed that the post-buckling behavior of symmetric laminate was completely different when the applied shear directions was alternated for the plate under either shear or combined loading. Kosteletos [5] investigated the post-buckling response of flat rectangular laminates with clamped edges under in-plane shear load or under combined in-plane loading, and the nonlinear von Karman type governing equations in terms of lateral deflection and stress functions were solved by the Galerkin method. Mittelstedt et al. [6] evaluated the post-buckling behavior of rectangular orthotropic laminated composite plates with initial imperfections under in-plane shear loading by the variational method. Beerhorst et al. [7] studied the post-buckling behavior of an infinitely long symmetric laminates with elastically torsional springs along the longitudinal edges under compression and shear using the similar method [6]. Based on the finite element method, Noor and Peters [8] presented the post-buckling analysis of laminated anisotropic plates subjected to compression and shear using a reduced basic technique and a problem-adaptive computational algorithm. Singh and Kumar [9] studied the post-buckling behavior of rectangular laminates under the action of in-plane shear loads. Subsequently, they solved the

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same problem under the action of uniaxial compression combined with in-plane shear loads [10]. More recently, Singh and Kumar [11–14] investigated the nonlinear behavior of composite laminates with various shaped cutouts under in-plane shear or under combined action of uni-axial compression and in-plane shear loads. Gupta et al. [15] studied the post-buckling behavior of laminated plates considering the geometric nonlinearity and evolving material damage under uniaxial, biaxial compressive and in-plane shear loadings. Han and Lee [16] evaluated the post-buckling behavior of laminated composite plates under the combination of in-plane shear, compression and lateral loading using an element-based Lagragian formulation, in which the natural strains, stresses and constitutive equations are used in the shell element. Wang and Dawe [17] presented an example of post-buckling behavior of a square, 8-layer, symmetric laminate under the edge shear using the spline finite strip method.

According to the authors' knowledge, there are no studies available on the nonlinear analysis of laminate plates under shear loading or combination of shear and compression loading using the finite strip method (FSM). In this paper, the finite strip method is developed to solve the post-buckling problem of the laminated composite plates under a combination of shear (via the skewed angle strain) and compression (via end shortening strain) loading. The validity of the method is illustrated by comparing the predictions with the finite element analysis, followed by a parametric study.

## 2. Theory

Consider a laminated composite plate with length of $a$ and width of $b$ (Fig. 1); the plate under a combination of in-plane shear and longitudinal compression loading can be regarded as a combination of the plate under the in-plane shear loading controlled by the skewed angle strain $\gamma$ (Fig. 1a) and the plate under the longitudinal compression loading controlled by the end shortening strain $\varepsilon$ (Fig. 1b), respectively. The transverse edges at $y=0$ and $a$ hold straight when the plate loaded in the pure shear; while the transverse edges are prevented from expanding laterally when the plate only loaded in the longitudinal compression.

The plate under shear loading via the skewed angle strain $\gamma$ can be obtained by applying the edge shear displacements $\alpha b/2$ and $\beta a/2$. When $\alpha=0$ and $\beta=\gamma$, the plate is subjected to the edge shear displacements $\gamma a/2$ only on the transverse edges; when $\alpha=\gamma$ and $\beta=0$, the plate is loaded with the edge shear displacements $\gamma b/2$ on the longitudinal edges only; finally when $\alpha=\gamma/2$ and $\beta=\gamma/2$, the plate is under the edge shear displacements $\gamma b/4$ and $\gamma a/4$ on the longitudinal and transverse edges, respectively.

Consider a typical finite strip element based on the First Order Shear Deformation Plate Theory (FSDPT), the displacements of the middle surface of the plate $u'(x,y)$, $v'(x,y)$ and $w'(x,y)$, and rotations of the normal to the middle surface $\phi_x'(x,y)$ and $\phi_y'(x,y)$ under combined compression and shear loading can be expressed as

$$\begin{aligned} u'(x,y) &= u \\ v'(x,y) &= \varepsilon\left(\frac{a}{2}-y\right)+\gamma\left(x-\frac{b}{2}\right)+v \\ w'(x,y) &= w \\ \phi_x{}'(x,y) &= \phi_x \\ \phi_y{}'(x,y) &= \phi_y \end{aligned} \tag{1}$$

In the finite strip analysis, the displacements $u$, $v$ and $w$ and the rotations $\phi_x$ and $\phi_y$ are expressed by the interpolation polynomial function in $x$-direction and smooth series functions in $y$-direction:

$$\begin{aligned} u &= \sum_{m=1}^{r} Y_u^m\ \{C_u\}\{\delta\}_m^e \\ v &= \sum_{m=1}^{r} Y_v^m\ \{C_v\}\{\delta\}_m^e \\ w &= \sum_{m=1}^{r} Y_w^m\ \{C_w\}\{\delta\}_m^e \\ \phi_x &= \sum_{m=1}^{r} Y_{\phi_x}^m\{C_{\phi_x}\}\ \{\delta\}_m^e \\ \phi_y &= \sum_{m=1}^{r} Y_{\phi_y}^m\ \{C_{\phi_y}\}\{\delta\}_m^e \end{aligned} \tag{2}$$

where $\{\delta\}_m^e$ is a vector representing the $m$th term nodal displacement parameters at the node lines of the finite strip element. For the low order finite strip with three node lines (LO3, see Fig. 2) [18,19], the following expression is held:

$$\{\delta\}_m^e = \{u_{im}\ v_{im}\ w_{im}\ \phi_{x\ im}\ \phi_{y\ im}\ u_{km}\ v_{km}\ w_{km}\ \phi_{x\ km}\ \phi_{y\ km}\ u_{jm}\ v_{jm}\ w_{jm}\ \phi_{x\ jm}\ \phi_{y\ jm}\}^{\mathrm{T}} \tag{3}$$

where $\{C_u\}$, $\{C_v\}$, $\{C_w\}$, $\{C_{\phi_x}\}$ and $\{C_{\phi_y}\}$ are the transverse interpolation shape functions, and they are given by

$$\{C_i\} = \{1\ x \ldots x^k\}[A_i], \quad i = u, v, w, \phi_x, \phi_y \tag{4}$$

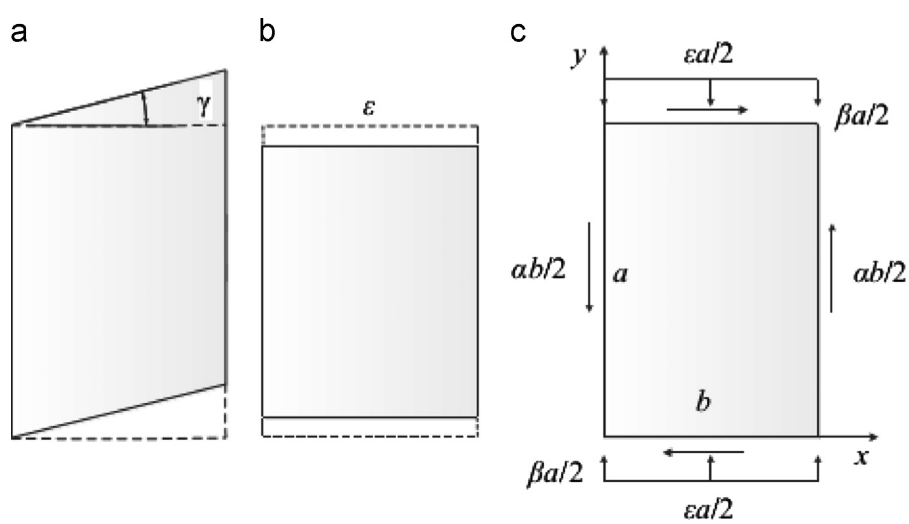


**Fig. 1.** The plate under the combination shear and compression loading: (a) shear action via skewed angle strain, (b) compression action via longitudinal end shortening, and (c) combined skewed angle strain and end shortening.