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Finite element analysis for buckling of two-layer composite beams using Reddy's higher order beam theory

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ABSTRACT

In this paper, a two-layer partial composite columns model is built based on Reddy's higher order beam theory, and two novel displacement based finite elements for this and Timoshenko composite beams are respectively formulated by means of the principle of minimum potential energy. Subsequently, the buckling analyses of pinned–pinned and clamped–guided composite columns are performed using the proposed finite elements, and the results are compared with those obtained by plane stress model, Timoshenko and Newmark composite beams model respectively. The superior quality of Reddy composite columns model over Timoshenko composite columns model and the correctness of the proposed Timoshenko composite columns model are demonstrated by the numerical comparison. Finally, the parametric study explores effects of parameters including stiffness of shear connectors, span-to-depth ratios, Young's modulus ratios and sub-layer's depth on the buckling load. The discrepancies between the performance of higher order and Timoshenko composite columns have also been numerically investigated.

1. Introduction

Composite layered systems are increasingly being employed in various engineering applications due to their optimized material configuration, high strength-to-weight and stiffness-to-weight ratios, and so on. For instance, steel-concrete composite beams are designed to make better use of the compressive strength of reinforced concrete slab and the high tensile properties of the steel joist. All the components of two-layer composite beams work together through the connection of shear connectors, thus, the shear connectors play a crucial role in the mechanical behavior of composite beams. Accordingly, models of layered composites, due to the rigidity of the shear connectors, can be grouped into at least three categories: (1) full composite beams model [1], which assumes rigid interfacial connection can be achieved; (2) partial composite beams model [2,3], which takes the interfacial slip into account and is still a popular and reasonable one, though the plane cross-section assumption is still in use; (3) the higher order shear deformable model [4,5], where higher order shear deformation of each sub-layer together with the partial interaction are considered.

Mechanical models of two-layer composite beams have been developing. Newmark et al. [6] as one of the pioneers studying the two-layer partial composite beams, formulated the governing differential equations for elastically connected steel-concrete beams, based on the linear elastic Euler–Bernoulli beam theory. Later on, their model was developed for the dynamic and non-linear problems [7,8]. Ranzi and Zona [3,9] modeled the reinforced concrete slab and steel joist with Euler–Bernoulli and Timoshenko beam theory (TBT) respectively, and both time-dependent and static analyses were carried out by the finite element method (FEM). Moreover, Ref. [10–13] have proposed or applied mechanical models considering the higher order shear deformation, of which Reddy's [11] theory is one of the most popular ones. This theory assumes that the axial displacement of beam varies as cubic polynomial over the beam depth, as a result, the parabolic distribution of shear flow over beam depth can be achieved. Thus, there is no need to introduce the shear corrector factor used by TBT, which is one of the appealing merits. That's because it is a tough problem [14] to obtain Timoshenko's shear correction factor for composite beams with partial interaction, as the factor is attributed to each component cross-section's geometry as well as the shear stress around the section [4]. Recently, Reddy's higher order beam theory (RHBT) has been paid close attention to the study of composite beams. Chakrabarti et al. [4,5,15], for example, have proposed the finite elements for two-layer linear elastic partial composite beams, subsequently analyzed both static and dynamic responses of these composite beams by FEM.

A considerable amount of research has been conducted on the stability of composite structures. However, most of the literatures

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on buckling analyses of two-layer composite beams seem to be based on the classic beams theory: Euler–Bernoulli beam theory (EBT) and/or TBT. Even very recently, there are still plenty of reports [16–22] on these. Although there are several reports [4,5,15] aforementioned on RHBT composite beams, most of them are confined to static or dynamic response analyses. To our knowledge, there seems to be no analysis in the open literature for buckling analysis of RHBT partial composite columns, which motivates us to investigate the buckling characteristics of RHBT composite columns and complement the aforementioned studies of Refs. [4,5,15].

To overcome the drawbacks about shear correction factor, the present paper performs the buckling analysis of RHBT composite columns, and a novel displacement based locking free finite element is formulated, by means of the principle of minimum potential energy. This finite element (FE), where both Lagrange and Hermite polynomials are employed to construct the shape functions, is composed of three nodes, and each node contains 6 degrees of freedom (DOF), so as to avoid the introduction of penalty coefficient [23], which may cause numerical problem [23]. The reliability and quality of the proposed finite elements are verified through comparisons among the solutions of the present, plain stress model, EBT model, simplified TBT model of Xu and Wu [2] and TBT model of Grognec et al. [18]. In addition, FE formulation for partial TBT composite beams is given to study the discrepancy between TBT and RHBT. And full RHBT composite beams (whose shear connection is rigid) are also presented aiming to make a thorough comparison to examine the shear locking owing to the high stiffness of shear studs. In the parametric study, parameters including slenderness ratios, rigidity of shear connectors, Young's modulus ratios and sub-layer depth ratios are investigated.

2. Axial displacement hypotheses

As is shown in Fig. 1, coordinate x denotes the location of composite beams' cross-section; subscript c and s denote the component of layer c and s, respectively. The overall depth of the composite beams is divided by the slip interface and the two x axes at each sub-layer's centroid into four, which are h_1, h_2, h_3, and h_4 from top to bottom. Symbols u_{c0} and u_{s0} in Fig. 1 indicate the axial displacement at cross-section's centroid of component c and s respectively; θ_c and θ_s are the tangent slope of component c and s at centroid of cross-section, respectively. The interfacial slip is noted as u_{cs}. According to Reddy's [11] third order axial displacement assumption, the axial displacement mode can be expressed as

$$u_i(x,y_i)=u_{i0}(x)-\theta_i(x)\,y_i+\alpha_i(x)\,y_i^2+\delta_i(x)\,y_i^3,\quad i=c,\ s. \tag{1}$$

where u_i is the axial displacement of layer i; $\alpha_i(x)$ and $\delta_i(x)$ are the coefficients for higher order terms. The process of determining $\alpha_i(x)$ and $\delta_i(x)$ is given below.

Neglecting the uplift between the two components, i.e. the transverse displacement of each sub-layer is the same. Thus, the longitudinal interfacial slip u_{cs} and transverse deflection of composite beams w can be written as

$$u_{cs}(x)=u_c(x,-h_2)-u_s(x,h_3) \tag{2}$$

$$w_c(x)=w_s(x)=w(x) \tag{3}$$

Shear strain and stress at cross-section can be obtained using the physical and geometric equations from theory of elasticity [24]. Thus, for part i, shear strain γ_i and shear stress τ_i can be expressed as

$$\gamma_i=\frac{\partial u_i}{\partial y_i}+w' \tag{4}$$

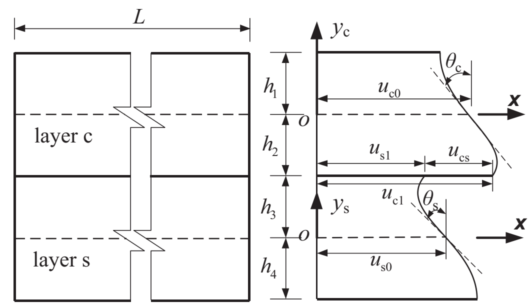

Fig. 1. Axial displacement assumption of RHBT composite beams.

$$\tau_i=G_i\,\gamma_i \tag{5}$$

where the prime denotes the derivative with respect to the x, i.e. $(\bullet)'=\mathrm{d}(\bullet)/\mathrm{d}x$; G_i is the shear modulus of layer i.

Assuming that there is no shear stress acting on the top and bottom surfaces of composite beams, in conjunction with the theorem of conjugate shearing stress, yields two stress boundary conditions

$$\tau_c(h_1)=0 \tag{6}$$

$$\tau_s(-h_4)=0 \tag{7}$$

The above two equations (6) and (7) are the same as Refs. [4,5,15] used, and this paper is going to formulate the other two constraints by the interface force continuity conditons between the two layers, rather than by the way of Refs. [4,5,15] to introduce two adjacent axial displacements at the interface as independent variables.

We assume that the shear resistance of shear stud is smeared uniformly on the interface, so that the interface force continuity conditons may be formulated as

$$b_c\,\tau_c(-h_2)=k_{cs}\,u_{cs} \tag{8}$$

$$b_s\,\tau_s(h_3)=k_{cs}\,u_{cs} \tag{9}$$

where the linear constitutive law of shear connectors is employed, i.e. $F=k_{cs}\ u_{cs}$, in which k_{cs} is rigidity of shear stud; F is the resistance when interfacial slip reaches u_{cs}; b_c is the width of cross-section c at the bottom; b_s is the width of cross section s on the top.

Moreover, the case of rigid shear connection can be formulated by imposing $u_{cs}=0$, that is

$$u_c(-h_2)=u_s(h_3) \tag{10}$$

$$b_c\,\tau_c(-h_2)=b_s\,\tau_s(h_3) \tag{11}$$

Higher order coefficients α_i and δ_i can be obtained by Eqs. (6)–(9) for partial RHBT composite beams, or by Eqs. (6), (7), (10) and (11) for full RHBT composite beams. Consequently, we have the explicit form of axial displacement field as

$$u_c=\mathbf{m}\,\mathbf{e} \tag{12}$$

$$u_s=\mathbf{n}\,\mathbf{e} \tag{13}$$

where $\mathbf{e}=\left[u_{c0}\quad \theta_c\quad u_{s0}\quad \theta_s\quad w'\right]^{\mathrm{T}}$; $\mathbf{m}$ and $\mathbf{n}$ are row vectors whose expressions are given in **Appendix A** in detail.

3. Finite element formulations

3.1. Geometric and physical equations

By using the physical and geometric equations [24], in conjunction with Eqs. (12) and (13), strain vectors can be expressed in