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Generalized shell heat transfer element for modeling the thermal response of non-uniformly heated structures

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ABSTRACT

A generalized shell heat transfer element is formulated in isoparametric coordinates to simulate the 3D thermal of non-uniformly heated shells with curved geometries. The element uses a combination of finite element and control volume methods to discretize the domain of the element into 2D layers that are coupled by a finite difference calculation. As demonstrated in previous work, the finite element-control volume formulation allows the thermal response to be evaluated with minimal computational expense and the temperature field is calculated in a manner that is compatible with distributed plasticity elements for structural analysis. Although the formulation uses a mixture of finite element and finite difference equations, the element equations are in a form that can readily be implemented in a commercial finite element code. The nine-node quadratic element considered here is implemented in Abaqus as a user-defined element. One-, two-, and three-dimensional verification cases are presented to demonstrate the capabilities of the element. Comparisons between the shell heat transfer element and traditional continuum heat transfer elements illustrate that the shell element converges rapidly and results in significant savings in computational expense.

1. Introduction

Macro finite elements (e.g., frame elements, shell elements, and component models) are widely used in computational mechanics to efficiently simulate structural response under various types of loading conditions. Macro elements utilize simplifying assumptions about the underlying physics to reduce the number of degrees of freedom in the model. For example, the Euler–Bernoulli hypothesis that plane sections remain plane during bending allows the behavior of a beam to be modeled by a line element with translational and rotational degrees of freedom at the element's ends. Due to the considerable reduction in degrees of freedom (i.e., in comparison to continuum models), macro elements allow the response of complex, large-scale structural systems to be studied in ways that would not be possible with continuum models due to excessive computational expense.

Despite the widespread use of macro elements in structural mechanics, limited research has sought to formulate macro finite elements for other field problems. Semi-analytical methods have been applied to efficiently calculate heat transfer in solids but these techniques lack generality. Surana and others [1–5] proposed a hierarchical *p*-version formulation to simulate the heat transfer in laminated shells. In the *p*-version formulation, the temperature field is interpolated from nodal values in the three coordinate directions. Continuity conditions are imposed at the interlaminar boundaries, allowing the temperature through the thickness of the shell to be approximated by a piecewise function. The lowest degree of polynomial that can be represented in the *p*-version formulation is linear, which requires a minimum of two nodes for each layer.

Jeffers [6] proposed an alternative formulation for a layered plate heat transfer element that was based on a combination of finite element and control volume approaches. As shown in Fig. 1, the layer temperature varied in plane according to the 2D shape functions, while the layers were treated as lumped masses in the transverse direction. To satisfy the principle of energy conservation, heat transfer in the transverse direction was simulated via a finite difference calculation. Verification exercises demonstrated that the element exhibited excellent accuracy with minimal computational expense, making the element an attractive alternative to 3D continuum elements.

The finite element-control volume approach for heat transfer analysis [6–8] has an added benefit of providing compatibility between the heat transfer model and distributed plasticity elements that are used in structural mechanics. The compatibility arises from the fact that there is a one-to-one mapping between

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the temperature degrees of freedom in the heat transfer element and the integration points in the structural element (i.e., for the calculation of thermal stresses/strains and the application of temperature-dependent constitutive relationships). The seamless transfer of data from the thermal analysis to the structural analysis prevents excessive calculations that would otherwise be needed to interpolate and/or discard temperature data.

In the present paper, the formulation by Jeffers [6] is extended to allow for shells of curved geometries, as illustrated in Fig. 2. The finite element-control volume approach is maintained in the present formulation. However, the element geometry is expressed in natural coordinates ξ, η, and ζ rather than in Cartesian coordinates x, y, and z. Thus, a geometric transformation is introduced in the formulation to transform the element from $\xi\eta\zeta$ coordinates to xyz coordinates. The generalized formulation is presented in Section 2, and a verification study is performed in Section 3.

2. Element formulation

The generalized shell heat transfer element (shown in Fig. 2) is discretized into n layers. It is assumed that the thickness of the shell is relatively small in comparison to the other dimensions (e.g., that its mechanical behavior can be described by shell bending theory). Because the thickness is small in comparison to the other dimensions, the temperature gradients produced by non-uniform heating on the surface of the shell are more pronounced over the thickness of the element than in other directions.

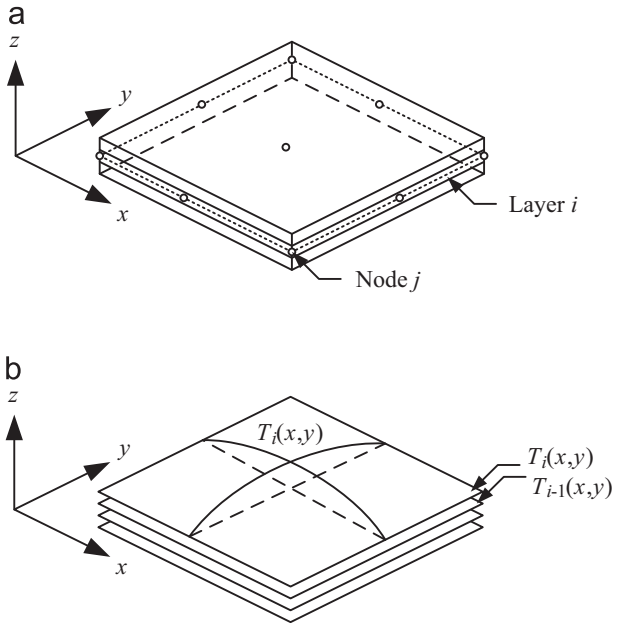

Fig. 1. Plate heat transfer element [6]. (a) Layers and nodes for the 9-node element, and (b) schematic of the temperature field in layer i.

Therefore, an optimal solution to the 3D heat transfer equations requires a large number of degrees of freedom over the thickness and fewer degrees of freedom to describe the in-plane behavior.

In the present formulation, the mixed finite element-control volume method proposed in [6–8] is used. The temperatures for each layer in the ξ–η plane are interpolated from the nodal temperature using 2D shape functions, while the layer temperatures in the ζ-direction are modeled as lumped masses. Governing equations in global xyz coordinates are transformed to the natural $\xi\eta\zeta$ coordinates based on a geometric relationship between the global and natural coordinates.

The degrees of freedom for the element consist of the layer temperatures at each node. For an element with n layers, the element has n temperature degrees of freedom at each node. If the element has m nodes, the element has a total of nm temperature degrees of freedom. Temperatures in the ξ–η plane may be interpolated as linear or quadratic. The present formulation considers a base geometry that is quadrilateral. However, the formulation can be extended to triangular elements.

2.1. Element geometry

The element's geometry is defined in terms of the nodal coordinates (x_j, y_j, z_j) at the mid-surface of the shell, the thickness t_j, and the thickness-direction vector $\mathbf{n}_j$ for each node j. Shown in Fig. 3, the thickness-direction vector $\mathbf{n}_j$ is dependent on the direction cosines l_j, m_j, and n_j of the line that is normal to the mid-surface, i.e., [9]

$$\mathbf{n}_j = t_j \begin{Bmatrix} l_j \\ m_j \\ n_j \end{Bmatrix} \tag{1}$$

If points 1 and 2 along the normal vector have coordinates (x_{1j}, y_{1j}, z_{1j}) and (x_{2j}, y_{2j}, z_{2j}), respectively, the direction cosines are calculated by

$$\begin{Bmatrix} l_j \\ m_j \\ n_j \end{Bmatrix} = \frac{1}{t_j} \begin{Bmatrix} x_{1j} - x_{2j} \\ y_{1j} - y_{2j} \\ z_{1j} - z_{2j} \end{Bmatrix} \tag{2}$$

Note that the midsurface coordinates are the average of the coordinates of points 1 and 2 at the top and bottom surfaces of the plate, i.e.,

$$x_j = (x_{1j} - x_{2j})/2$$
$$y_j = (y_{1j} - y_{2j})/2$$
$$z_j = (z_{1j} - z_{2j})/2 \tag{3}$$

Because the element geometry is described in this manner, the shape functions N_j are dependent on ξ and η only (i.e., the shape functions are independent of ζ). Thus, the coordinates of an arbitrary

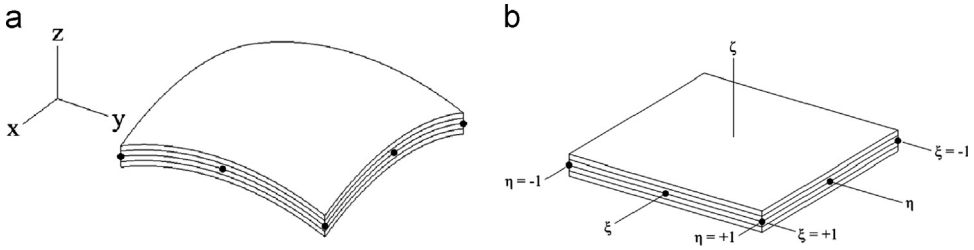

Fig. 2. Generalized shell heat transfer element: (a) in global coordinates, and (b) in natural coordinates.