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Numerical analysis for design optimization of microcantilever beams for measuring rheological properties of viscous fluid

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ABSTRACT

The precise measurement of rheological properties is a demanding problem in various field of engineering. Occasionally the available sample volume of interest may be sufficiently small where the conventional methods of measuring rheological properties are inappropriate. Consequently, there is a growing interest in the use of MEMS devices to measure the required properties, especially with an aim of encouraging high throughput. During this research, the dynamic response of micro cantilever beams is demonstrated to characterize the rheological properties of viscous materials. First, the dynamic response of a mini cantilever beam partially submerged in air and water is measured experimentally for different configurations using a duel channel PolyTec scanning vibrometer. Next, finite element analysis (FEA) method is implemented to predict the dynamic response of the same cantilever beam in air and water, and then compared with corresponding experiments. Once the model is validated, further numerical analysis is conducted to investigate the variation in modal response with changing beam dimension and fluid properties. Results obtained from this parametric study can be used for sensitivity analysis and to design the optimized MEMS based test set up for measuring the rheological properties of viscous fluid and of any soft viscoelastic materials such as biofilm. Miniaturization of the measuring instrument is necessary so that small sample volume can be used to perform the desired test.

1. Introduction

Viscosity and density of a fluid are two important parameters to characterize the fluid's behavior. Viscosity is often thought as the fluid's friction, resistance to flow or the fluid's resistance to shear when the fluid is in motion. It is often represented as a coefficient that describes the diffusion of momentum in the liquid. The measurement of viscosity has been employed for many decades to monitor and test lubricants, blood, mucus, adhesives, paint, fuels and other fluids.

In various fields of engineering, the measurement of fluid viscosity and density is a particularly demanding problem when precision is required. For many engineering applications, different types of rheometers are used, where these viscosity measurement techniques require both large experimental apparatus and sample volume. In this study, we explored the numerical method to predict the dynamic response of a mini cantilever beam submerged in viscous fluids. In this particular application, miniaturizing the measuring instrument is necessary due to the presence of microscale heterogeneity in the mechanical/rheological properties. In addition, occasionally the available volume of the liquid of interest may be sufficiently small where the conventional methods of rheometry such as cone and plate rheometry [1], stormer viscometry [2] or falling ball viscometry [3] are inappropriate. Consequently, there is a growing interest in the use of MEMS devices to measure the required properties, especially with an aim of encouraging high throughput. These devices include pressure sensors [4], optical tweezers [5] and micro-particle image velocimetry [6], and others [7–10].

The dynamic response of micro-cantilever beams is extensively used for different research aspects related to MEMS devices. Sader [11] studied the vibrational characteristics of a mini-cantilever beam immersed in viscous fluids. He presented a detailed theoretical analysis of the frequency response of a cantilever beam excited by an arbitrary driving force. Due to its practical importance in application to the atomic force microscope (AFM), he considered in detail the special case of a cantilever beam that is excited by a thermal driving force. This incorporated the explicit analytical formulae and numerical results, which were valuable to the users and designers of AFM cantilever beams.

Boskovic et al. [12] introduced the use of a single micro-cantilever beam to simultaneously measure the rheological properties of gases and liquids. The research employed the theoretical model of Sader [11] that dealt with frequency response of AFM cantilevers immersed in viscous fluids. First, the frequency responses of cantilever, resonant frequency ω_R and *Q*-factor, are determined from its deflection measured by optical method.

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The measured value of ω_R and Q-factor are then numerically solved using a root finding procedure to simultaneously determine the density and viscosity. The rheological properties of commonly used gases and liquids measured by Boskovic's model offered excellent matching compared to published data. The technique presented by Boskovic et al. only required knowledge of cantilever geometry, and its resonant frequency in vacuum, and its linear mass density. However, the model used to extract the fluid density and viscosity from the measured frequency response requires the quality factor $Q > 1$.

Belmiloud et al. [13] demonstrated that vibrating microcantilevers can be used to quantify fluid properties such as density and viscosity. They developed a microrheometer based on the measurement of fluid properties over a range of vibration frequencies, without necessarily be restricted to resonant phenomenon. They developed analytical model to express the dissipative and inertial parts of drag force exerted on microcantilever by viscous fluid. The determination of drag forces allowed extracting the density and viscosity of the unknown fluid.

Basak et al. [14] developed a 3D finite element based fluid–solid interaction model to predict the hydrodynamic loading of microcantilevers in viscous fluids. The quality factor and added mass coefficients of several modes were computed accurately from the transient oscillations of the microcantilever in the fluid. The effects of microcantilever geometry, operation in higher bending modes, orientation and proximity to a surface were analyzed. The study found that the main energy dissipation in a viscous fluid arises from the localized fluid shear near the edges of vibrating cantilever. Closer to a surface, however, the damping arises due to a combination of squeeze film effects and viscous shear. The quality factors and wet natural frequencies of different modes decreases upon approach to a rigid surface, while the added mass coefficient increases. It was also found that modifying the cantilever geometry can lead to unexpected changes in quality factor and wet natural frequencies. In order to maximize the quality factor and wet natural frequencies, the effective width of the cantilever needs to be maximized and the presence of slots and edges and the length need to be minimized. This 3D finite element model, which includes the fluid–solid interaction, was found a powerful tool to optimize the complicated microcantilever geometry applicable to atomic force microscopy and biosensors.

Basak et al. [15] also used 3D transient fluid–solid interaction model to study the single and dual axis torsional probes for atomic force microscopy in liquids. The computational analysis provided information on the mechanics of hydrodynamic dissipation important for torsional probes to be designed for optimal performance prior to microfabrication. For single axis torsional probes, the primary areas for improving performance are reductions in pad thickness and pad area. Simulations of dual axis probes showed that rotations about the orthogonal axes remain mostly independent, even at resonant frequencies. This coupling is mostly intrinsic to the mechanics of the structure. However, external hydrodynamics slightly influence the coupling.

Clark et al. [16] explored analytically, numerically and experimentally the spectral properties of the flexural vibrations of micron scale cantilevers in a viscous fluid that are driven externally or by Brownian motion. The analytical expression and the numerical approach discussed have a wide range of applicability that can be used to gain physical insights and guide future experiments related to micro and nanoscale technologies that exploit the high frequency oscillations of elastic objects in viscous fluids. Although there is a significant difference between driving a cantilever externally or using only thermal motion, they have developed a unified approach to quantify the dynamics that requires straight forward deterministic calculations.

Mather et al. [17] used a piezoelectric bimorph cantilever to assess the rheological properties of Newtonian and Non-Newtonian viscous fluids. Formulations were presented that relate the dissipative and internal parts of the drag force exerted by the fluid to the shear moduli. These shear moduli can be determined experimentally through the presented formulations, knowledge of the test fluid density and measured cantilever resonant frequency and quality factors. The cantilever displayed a sufficiently high quality factor to probe properties of highly damping and elastic fluids in situ. The frequency dependence of fluid viscoelastic properties could be investigated by studying the resonant response of multiple modes of cantilever or by studying the response of multiple cantilevers of different lengths.

Hass et al. [18] studied the steady-state shear viscosity and the linear viscoelastic behavior of dilute, semidilute, and densely packed vesicle dispersions as a function of shear rate and temperature. These experiments were supported by dynamic light scattering experiments and freeze-fracture electron micrographs. The experiments were conducted by varying the volume fraction at constant temperature and temperature at constant volume fraction. The findings were compared with experimental results reported in literature for unilamellar and multilamellar vesicles.

Van Eysden and Sader [19] studied the dynamic response of cantilever beams immersed in compressible fluids to investigate the significance of fluid compressibility. A rigorous theoretical model was presented for the frequency response of a rectangular cantilever beam executing normal and torsional oscillations. It was found that compressibility becomes increasingly important as the mode number rises. Martin and Houston [20] analyzed the dynamic response of different micro-scale silicon cantilevers in air and liquid using an integrated fluid–structure solver. Bode diagrams and Nyquist plots of the cantilever transfer function indicated the resonator as a heavily damped system in liquid, and a lightly damped system in air. Riesch and Kepliger [21] developed an analytical model to characterize resonating cantilevers for sensing liquid properties. The analytical model allowed the fluid–solid interaction between the liquid and the oscillating beam of different cross-sections. The analytical model was based on an approximation of the immersed cantilever as an oscillating sphere comprising the effective mass and the intrinsic damping of the cantilever, and additional mass and damping due to the liquid loading. The model parameters were obtained by a curve fitting procedure. Herruzo and Garcia discussed [22] the dynamics of an amplitude modulation atomic force microscope in different environments such as water and air. Experiments, analytical expressions, and numerical simulations showed that the resonance curves depend on the excitation method used to drive the cantilever. Under mechanical excitation, the deflection involves the base and tip displacements, while in magnetic excitation, the cantilever deflection and tip displacement coincide. Dalessandro and Rosato [23] numerically studied the frequency response of an electromechanical system made of a metallic cantilever hosting piezoelectric lamina working in both sensor and actuator modes. In order to calculate the interfacial mechanical stresses, they have used a 3-D model that was able to take into account the coupling between longitudinal, flexural, and torsional modes, resulting in a more accurate reproduction of the physical system, which was sufficient for studying the debonding problem or for a pointwise numerical assessment of the quantities of interest. Hossain and Mian [24] and Hossain et al. [25] employed the finite element analysis (FEA) method to predict the dynamic response of a mini cantilever beam partially submerged in viscous media. The beam and viscous media were simulated using the solid and fluid elements, respectively. The FEA method was validated by comparing the numerical results with corresponding experiments. An excellent agreement was found between the numerical and experimental results.

The layout of this paper is as follows. First, the experimental response of a mini cantilever beam in air and water is presented for completeness of this paper. Details of the experimental