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Parameter free structural optimization applied to the shape optimization of smart structures

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ABSTRACT

This contribution applies structural shape optimization techniques to actuated smart structures. In order to combine these two disciplines, the idea of Optimal Actuation and an enlarged filtering scheme are presented. A parameter free optimization scheme is used, which allows the treatment of different branches of structural optimization using an identical framework.

1. Introduction

The topic of structural optimization is established in research as well as in the industry. Usual applications target an optimal use of material, where the objective of optimization is minimizing structural weight while complying with displacement or stress constraints, or minimizing structural deformations for a given weight of the structure. One very famous branch of optimization where exactly these kinds of problems are treated is the field of topology optimization [1], but these questions also arise quite frequently in the fields of shape optimization and sizing [3]. Well established applications usually focus on “classical” structures, which are only affected by a given external load. In contrast to these structures, the so-called smart structures are additionally equipped with actuators, which can produce additional mechanical input in order to modify the structure's response. Actuators are usually controlled by optimal control algorithms [4,5], which focus on generating the desired structure response by superposition of the external load and the actuator action.

In this context, this contribution focuses on applying structural optimization in order to generate structures which behave optimally with respect to actuating input, whereas it does not focus on the topic of control and controller design.

Nowadays publications regarding optimization with respect to smart structures mainly focus on actuator placement for a given host structure using stochastic methods [6] or topology optimization [7]. As most applications of smart structures belong to the aerospace industry, a lot of publications concerning optimization do so as well. Farhan Gandhi and others are doing a lot of research work about morphing airfoils, where for example sizing optimization is applied to compliant mechanisms using honeycomb structures [8]. Bilgen and Friswell [9,10] are working on a variable-camber wing for an unmanned light weight aircraft, which is actuated by piezoelectric components. The wing was optimized in order to obtain an ideal lift-to-drag ratio in the actuated state. Therefor a Genetic Optimization Algorithm was applied on a parameterized optimization model, using half a dozen of design parameters. Although this work implied means of structural optimization, the focus was on experimental testing.

This contribution aims on structural optimization of smart structures without the necessity of introducing an additional parameterized optimization model, in combination with gradient based optimization methods. The focus is on generating structures which provide an optimal response to the actuator input.

A novelty of this contribution is that shape optimization involving a large design space is applied to smart structures, where especially free shape optimization of thin structures is a central aspect. By this mean, new designs for smart structures can be discovered, which can hardly be reproduce using a parameterized optimization model. The topic of actuator placement is also added to the parameterization model free optimization approach, such that costs of actuation can be measured, evaluated

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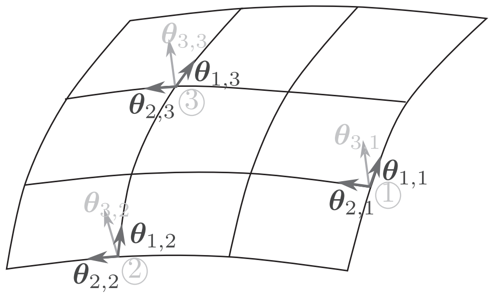

Fig. 1. Example of finite element based definition of shape determining design variables.

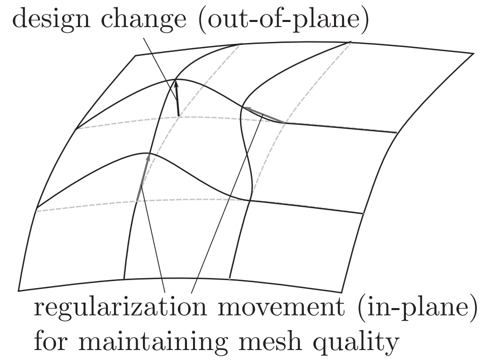

Fig. 2. Design modification and regularization movement.

and considered in the optimization. Thus structural optimization and actuator placement can be combined, and smart structures can be optimized with respect to a most efficient actuation.

2. The concept of parameter free shape morphing

For numerical treatment of problems in structural optimization, adequate parameterization of the design space is necessary. A meanwhile classical way of parameterizing is to apply an extra CAD-model as a design model [11]. The idea of using NURBS or B-Rep information in order to parameterize an optimization model is state of the art [12].

In contrast, the actual contribution uses a node based parameterization approach. This approach defines optimization parameters directly on the finite element mesh which is used for structural analysis. A separate model for design parametrization is not necessary. Therefore, the name "parameter free" in the sense of "free of CAD parameters" became conventional. Now, the parameters for shape design are the coordinates of each finite element node or vertex in context of this paper [23,20].

Fig. 1 shows an example of a finite element mesh for a curved surface, where for three nodes design parameters for shape optimization are visualized. For this purpose, at each node i a tangential coordinate system with the base vectors $\boldsymbol{\theta}_{1,i}, \boldsymbol{\theta}_{2,i}, \boldsymbol{\theta}_{3,i}$ is defined, with $\boldsymbol{\theta}_{3,i}$ being the normal vector of the surface. The coordinates $t_{3,i}$ in normal directions $\boldsymbol{\theta}_{3,i}$ are considered design parameters for shape modification. At a regular node in the interior of the surface, a tangential movement of the node affects the mesh quality, but not the shape. Therefore, tangential coordinates $t_{1,i}$ and $t_{2,i}$ are adapted by secondary arguments to guarantee good mesh quality. Regularization techniques for shape quality control in shape optimization were developed by eg. [16,24,18,20] or [17]. Fig. 2 shows a one-node out-of-plane design change and the related in-plane regularization movements.

Knowledge about surface normal directions allows us to compute the sensitivity p_i of a response function f with respect to the shape modification at a considered node i by computing the total differential of the response in direction $\boldsymbol{\theta}_{3,i}$:

$$p_i = (\nabla f)^T \cdot \boldsymbol{\theta}_{3,i} \tag{1}$$

Here, ∇f is to be understood as the derivative of f with respect to the nodal shape parameters which are the three spacial coordinates x_i of node i. The term p_i is identified as the discrete equivalent at node i of the continuous shape derivative. Mathematical literature about computation of shape gradients and shape calculus can be found in [13–15].

3. Mathematical formulation of shape optimization and shape control

The goal of this chapter is to present the underlying theory of the node based vertex morphing method using discrete nodal parameters. This way, we start from a continuous formulation and will discretize later on.

We consider a surface Ω in space and related surface coordinates $\xi \in R^2$ which eventually are called material coordinates as well. The surface geometry is given by the spacial coordinates $x = x(\xi)$ in R^3. The field $t_3 \in R$ is the field of coordinates in normal direction at every surface point ξ. The normal defines the direction of shape evolution at surface point ξ. Additionally, we consider a control field $s(\xi) \in R$. The normal coordinate t_3 at a surface point ξ_i is linked to s through the kernel filter function $F(\xi, \xi_i)$ and the convolution integral over the surface Ω:

$$\begin{aligned} t_3(\xi_i) &= \int_\Omega F(\xi, \xi_i) \cdot s(\xi)\, d\xi \\ \int_\Omega F(\xi, \xi_i)\, d\xi &= 1 \end{aligned} \tag{2}$$

The shape optimization is driven by the manipulation of the control field s.

To that end we define the shape optimization problem as:

$$\begin{aligned} &\min f(s,u) \\ &\quad u = u(s) \quad \text{from } S(s,u) = 0 \end{aligned} \tag{3}$$

where f is the objective as a function of the control field s and u which are the fields of state variables, e.g. the displacement fields in R^3, and S is the state equation.

The derivative of the objective f with respect to the design control field s is determined by adhering to chain rule of differentiation:

$$\frac{df}{ds} = \int_\Omega \frac{df}{dt_3} \cdot \frac{dt_3}{ds}\, d\xi = \int_\Omega p \cdot \frac{dt_3}{ds}\, d\xi \tag{4}$$

where the shape derivative p is the derivative of the objective with respect to the normal coordinates considering shape variations in the surface normal direction. Furthermore, evaluation of $\frac{df}{ds}$ at position ξ_i yields

$$\frac{df}{ds}(\xi_i) = \int_\Omega p(\xi) \cdot F(\xi_i, \xi)\, d\xi \tag{5}$$

We observe that derivatives are treated with the transpose filter operation as compared to the filtering of the normal coordinates t_3. Of course, in the case of symmetric filter functions it holds

$$F(\xi_i, \xi) = F(\xi, \xi_i) \tag{6}$$

Typically, a piecewise linear hat-function of C_0-continuity is used as a filter kernel function $F(\xi, \xi_i)$. This function can be