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Transformed perturbation stochastic finite element method for static response analysis of stochastic structures

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ABSTRACT

To obtain the probability density functions and the cumulative distribution functions of static responses of stochastic structures, a hybrid stochastic method named as the transformed perturbation stochastic finite element method (TPSFEM) is proposed. In TPSFEM, the static responses of stochastic structures are approximated as the linear functions of random variables by using the first order perturbation technique. According to the approximated linear relationships between static responses and random variables, the probability density functions of static responses are obtained by the change-of-variable technique. The cumulative distribution functions of static responses are calculated by the numerical integration method. The numerical examples on a thin plate, a six-bar truss structure, a Mindlin plate and a shell structure verify the effectiveness and accuracy of the proposed method. Hence, the proposed method can be considered as an alternative engineering method for the static response analysis of stochastic structures.

1. Introduction

Due to the effects of model inaccuracies, physical imperfections and system complexities, uncertainties widely exist in engineering structures including bridges, buildings, offshore structures, vehicles, ships, aerospaces and so on. In most cases, the uncertainties existing in engineering structures can be treated as random variables whose probability density functions are defined unambiguously based on the statistical analysis of sample data. The uncertain structures with random variables can be described as stochastic problems. Probability approaches can be considered as the most feasible techniques for stochastic problems. In engineering practices, the use of the random theory in the finite element method (FEM) has initiated the development of the stochastic finite element method (SFEM) [1–3]. SFEM has received considerable attention in the last several decades. The ability of SFEM to treat with the non-deterministic properties is of great value for a design engineer realizing the reliability assessment and the robustness analysis of uncertain structures.

Monte Carlo method is the most versatile probabilistic method for stochastic problems. According to the probabilistic convergence, the accuracy of Monte Carlo method strongly depends on the number of samples generated by a random number generator. With the increase in the number of samples, the accuracy of Monte Carlo method is improved gradually. However, the improvement in the accuracy of Monte Carlo method is always at the cost of computational burden. Thus, Monte Carlo method cannot be directly applied to large-scale engineering problems. In order to reduce the computational cost of Monte Carlo method without deteriorating its accuracy, numerous variants (the importance sampling method [4], the subset simulation method [5], the line sampling method [6] and the other extended methods [7]) have been developed in the last decade. Each variant has its merits and demerits [8]. Monte Carlo method and its variants are random sampling method. Recently, some non-random sampling method, such as Metropolis sampling method [9,10] and interval sampling method [11,12], have been developed. The main advantage of these non-random sampling methods is that the total number of samples can be reduced greatly. However, the computational accuracy of these non-random sampling methods is depended on the sampling rule which is formulated on the basis of sample model. If the sampling rule is inappropriate, results yielded by the non-random sampling methods may be deviated from the real ones seriously. Therefore, up to now, Monte Carlo method is still considered as the most robust probabilistic method and is usually used as a reference method to investigate the accuracy of other probability approaches [13–15].

Spectral stochastic method, in which the random input parameters are modeled by Karhunen–Loève formulation and the random response quantities are approximated by the polynomial

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chaos expansion, is an alternative approach for stochastic problems [16]. The application of the spectral stochastic method within the finite element framework has initiated the development of the spectral stochastic finite element method (SSFEM) [17–21]. The main advantage of SSFEM is that the complete probability distributions of random variables can be provided. According to the researches on the convergence and accuracy of SSFEM [22,23], we can obtain that the results estimated by SSFEM converge to the "exact" solutions when the number of terms in the polynomial chaos expansion increases towards infinity. Unfortunately, with the increase in the number of terms, the computational burden of SSFEM increases sharply. This is why the application of SSFEM has been limited to the stochastic systems with a small number of degrees of freedom in the past time. Furthermore, the application range of SSFEM is strongly depended on the approaches adopted to calculate the coefficients of expansion.

Perturbation stochastic finite element method (PSFEM) is another important branch of stochastic methods [24–33]. In this approach, the random system matrices and the random loading vectors are expanded by the Taylor series. As the improvement in precise obtained by using the higher order approximations is rather small when compared to the disproportional increase of the computational complexity, the first order approximation is the most feasible strategy [34]. Based on the first order perturbation analysis, the responses of stochastic structures are transformed into the linear functions of random variables. By linearizing the relationships between the responses of stochastic structures and random variables, the computational complexity of PSFEM is simplified and the mathematical formulae of PSFEM can be easily implemented in the standard finite element codes. In most cases, the results obtained by PSFEM are expressed as the expectations and standard deviations of response vectors [25,27–33]. The expectations and standard deviations are the important mathematical characteristics of random variables. In the probability theory, if the types of the probability distributions of random variables are not defined unambiguously, the probability density functions of random variables cannot be determined even though the precise values of expectations and standard deviations are obtained. Consequently, the classical PSFEM cannot be applicable to determine the probability density functions of random responses of stochastic structures, unless the random responses can be approximated as Gaussian random fields [25,26]. As a matter of fact, the uncertain parameters in many cases cannot be modeled as Gaussian variables. If some uncertain parameters are modeled as non-Gaussian variables, the random responses are usually strongly non-Gaussian. Therefore, how to effectively apply the classical PSFEM to the stochastic structures whose responses cannot be approximated as Gaussian random fields is still a challenge needing to be dealt with.

In probability theory, the change-of-variable technique is used to derive the probability density functions of dependent variables according to the probability density functions of independent variables and the mapping relationships between dependent variables and independent variables [35]. In application of the change-of-variable technique, the mapping relationships between dependent variables and independent variables should be invertible. Unfortunately, in the practical engineering, the responses of stochastic structures are not likely to be the invertible functions of random variables. Even if the responses of stochastic structures are the invertible functions of some random variables, the mapping relationships between responses and random variables may be too complicated to obtain the corresponding inverse functions.

As is mentioned above, the mapping relationships between responses and random variables of stochastic structures can be linearized by the classical PSFEM based on the first order perturbation analysis. However, the classical PSFEM cannot be used to calculate the probability density functions of responses unless the responses can be approximated as Gaussian random fields. The change-of-variable technique can be used to calculate the probability density functions of dependent variables, if the probability density functions of independent variables are defined ambiguously and the mapping relationships between dependent variables and independent variables are invertible. Thus, a hybrid stochastic method which integrates the classical PSFEM and the change-of-variable technique within a unified framework may be developed to calculate the probability density functions of responses of stochastic structures.

In this paper, a unified framework integrating the classical PSFEM and the change-of-variable technique together is constructed. Based on the constructed unified framework, a hybrid stochastic method named as TPSFEM is proposed for the static response analysis of stochastic structures. In the proposed method, the responses of stochastic structures are simplified to the linear functions of random variables based on the first order perturbation analysis. The invertible function of a linear function can be easily obtained by some simple mathematical operations. Thus, the change-of-variable technique can be effectively applied to calculate the probability density functions of responses of stochastic structures. TPSFEM for the static response analysis of stochastic structures requires the following steps: (1) expanding the random stiffness matrices and the random loading vectors by using the first order Taylor series; (2) calculating the static responses of stochastic structures based on the first order perturbation analysis; (3) determining the coefficient vectors of the corresponding random variables; and (4) approximating the probability density functions of the static responses of stochastic structure based on the change-of-variable technique and calculating the cumulative distribution functions of the static responses of stochastic structures based on the numerical integration method.

The organization of this paper is listed as follows. The mathematical backgrounds about the numerical methods based on the change-of-variable technique to calculate the probability densities of the function with a random variable and the function with two random variables are discussed in Section 2. TPSFEM for the static response analysis of stochastic structures is deduced in Section 3. In Section 4, four numerical examples including a thin plate, a six-bar truss structure, a Mindlin plate and a shell structure are provided to verify the effectiveness and accuracy of the proposed method. In Section 5, some conclusions are given.

2. Mathematical backgrounds

2.1. Probability density of a function with a random variable

Suppose $g(x)$ is a function of the random variable x, defined as

$$y = g(x) \tag{1}$$

where the probability density function of x can be expressed as $f_x(x)$.

By using the change-of-variable technique, the probability density function of y can be determined in the term of $f_x(x)$

$$f_y(y) = \frac{1}{|\mathrm{d}y/\mathrm{d}x|} f_x(x) = \frac{1}{|\mathrm{d}y/\mathrm{d}x|} f_x(g^{-1}(y)) \tag{2}$$

where $f_y(y)$ is the probability density function of y; $x = g^{-1}(y)$ is the inverse function of $y = g(x)$. Unfortunately, it may be difficult or even impossible to obtain the inverse function $x = g^{-1}(y)$, as not all functions have the corresponding inverse functions. If the inverse function $x = g^{-1}(y)$ cannot be obtained, Eq. (2) has little meaning. To overcome this disadvantage, a numerical method based on the Taylor expansion will be proposed in this section.