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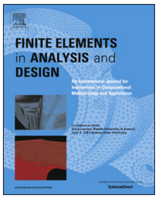

Eulerian analysis of bulk metal forming processes based on spline-based meshfree method

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ABSTRACT

An effective Eulerian approach employing Non-Uniform Rational B-Spline (NURBS) curves as a boundary representing technique is presented. This approach is based on the Spline-Based Meshfree Method: analysis domain boundaries are represented by NURBS curves and a NURBS patch is employed as a background mesh. By doing so, clear and neat boundary representations are possible. Moreover, the explicit definition of the analysis domain boundary dispenses with the need to solve convective transport equations. In this work, utilizing the above idea, bulk metal forming processes are treated within Eulerian framework based on the flow formulation of rigid-plastic materials. The implementation issues of the proposed method are discussed in detail. The effectiveness of the present approach is shown by comparing with other numerical methods. It is demonstrated that the method yields reliable results even in the case where severe deformation occurs.

1. Introduction

In the finite element analysis of metal forming processes, large deformation occurs accompanying severe mesh distortions. When dealing with such large deformation problems, conventional Lagrangian finite element analyses may have difficulties due to mesh distortion and therefore frequent remeshings are necessary to proceed the analyses. The remeshing increases computational cost and transfers solution parameters in an approximate way which can erode the solution accuracy. Vast amount of research can be found about the remeshing techniques from the 1970s, and these efforts led to the development of reliable commercial programs which are currently employed in metal industry.

Besides above-mentioned efforts, some researchers are still finding ways to eliminate the mesh distortion problems. Meshfree methods are proposed in this regard and also applied to the analysis of metal forming problems. For an overview of meshfree methods, please refer to [1–3]. Numerous applications of meshfree methods to metal forming problems can be found: such as reproducing kernel particle methods (RKPM) [4–9], smooth particle hydrodynamics methods (SPH) [10], element free Galerkin methods (EFG) [11,12], and least-squares meshfree methods (LSMFM) [13,14] These methods successfully analyzed metal forming processes dealing with the problems of large deformation effectively.

Arbitrary Lagrangian Eulerian (ALE) method is another approach to analyze the problems with large deformation. In ALE framework, the finite element mesh does not need to be attached to the material or be fixed in space. The mesh can be moved arbitrarily relative to the material. The method combined the strengths of Lagrangian framework and Eulerian framework and analyzed metal forming problems as well [15–18]. However, controlling the mesh motion complicates the methods and mesh distortion problems still can exist when dealing with severe distortion of complex geometries.

Another approach is the method based on Eulerian framework. In Eulerian framework, the mesh is fixed in space and only the material moves. Therefore, this framework is widely employed in fluid mechanics area where the deformation of material is severe. In the analysis of metal forming process, plastic deformation is dominant while elastic deformation is negligible. Therefore, the idealization to rigid-plastic material behavior is acceptable. The analysis based on this assumption is known as *flow formulation* [19]. In the flow formulation, metal workpieces are treated as an incompressible viscous non-Newtonian fluid. Several researches of the analysis of metal forming processes based on Eulerian framework can be found [20–26]. One of the main issues when implementing Eulerian framework is how to define and update the free surface of the domain. To define the free surfaces, several methods are proposed. One method is the pseudo-concentration method proposed by Thompson [27] and its applications to metal forming processes can be found [21–23]. The pseudo-concentration method assigns a pseudo-concentration variable (c_i) to each

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node. Then, the pseudo-concentrations function is interpolated inside the analysis domain: $c = N_i c_i$, where N_i is the basis function used in the analysis. The free surface is defined by the surface of $c = 0$, void regions are defined by $c < 0$ and material regions are defined by $c > 0$. During the analysis, pseudo-concentration variables and solution parameters are updated by solving convective transport equations. Another method is the volume of fluid (VOF) method proposed by Hirt and Nichols [28] and its applications to metal forming processes can be found recently [24–26]. The VOF method defines a function f which represents a fractional volume of the fluid (solid) in each element that has a value between zero (empty) and one (fully occupied). When a certain element has a value between zero and one, it means that the element contains a free surface. During the analysis, the function f and solution parameters are updated by solving convective transport equations.

The spline-based meshfree method (SBMFM) [29] is a new class of meshfree method based on isogeometric analysis [30]. The method combines the strengths of isogeometric analysis and meshfree methods. In the SBMFM, the non-uniform rational B-spline (NURBS) patch is employed as a background mesh and the boundaries of the analysis domain are explicitly defined by NURBS trimming curves on the NURBS patch. One of the distinguishing features of the SBMFM is that domain boundary update is not performed on DOFs, but on the curves. Therefore only the curves are updated based on numerical solutions (displacements or velocities) while background mesh remains. Since the curves and the background mesh are defined with complete independence, the curves can move without any restrictions. The prescribed feature is illustrated in Fig. 1. In the previous researches of the SBMFM [29], geometrically nonlinear problems are successfully treated.

In this work, the SBMFM will be applied to the rigid-plastic analysis of bulk metal forming processes. The analysis procedure can be roughly summarized as follows: (i) set up an analysis model using NURBS trimming curves and NURBS background mesh, (ii) perform a rigid-plastic analysis and obtain a velocity solution, (iii) update the model (NURBS curves) according to the velocity solution, (iv) march the time steps until the end of process. It is expected that the use of NURBS curves to define free boundaries will bring some benefits compared to other methods. First of all, the boundaries are explicitly represented without any smoothing or approximation processes. Second, the analysis domain boundaries are represented by the curves independently from the background mesh. As a consequence, domain update can be done with Lagrangian sense without solving convective transport equations, say $x_i^{t+\Delta t} = x_i^t + v_i \cdot \Delta t$. The circumvention of convective transport equations can drastically improve the convenience of the proposed method since upwind techniques such as SUPG method [31] are not necessary. Also, complex geometries can be represented regardless of the resolution of background mesh. In pseudo-concentration method or VOF method, the geometric resolution of analysis domain boundaries is determined by background mesh since the geometric data is stored in background mesh (pseudo-concentration method: nodes, VOF method: elements). Therefore, fine background mesh or certain smoothing techniques were necessary to obtain smooth and fine geometries. In the proposed method, the geometries of NURBS trimming curves are defined by control points of the curve, and any information of background mesh is not involved at all.

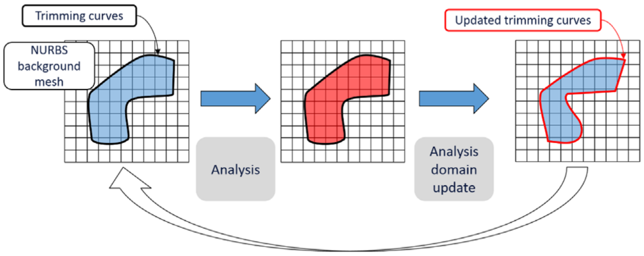

Fig. 1. Analysis framework of the SBMFM: domain boundary update is performed on NURBS trimming curves.

The remainder of this paper is organized as follows. In Section 2, the preliminary studies of the SBMFM is briefly introduced. In Section 3, the rigid-plastic formulation employed in this work is presented and solution strategy is presented. The implementation issues of this method, such as NURBS trimming curve update from velocity solution and time integration of effective strain are discussed in detail. Numerical examples are presented in Section 4. It is shown that the analysis is successfully performed even in the case with severe distortions. Finally, conclusions are given in Section 5.

2. A brief introduction of spline-based meshfree method

In this section, the background of the SBMFM is briefly introduced. The SBMFM is exploited from the trimmed surface analysis [32], and the trimmed surface analysis is a method to improve the flexibility of isogeometric analysis.

2.1. Isogeometric analysis

Isogeometric analysis [30,33] is a framework that integrates Computer Aided Design (CAD) and Finite Element Analysis (FEA) through the unification of the basis functions of the both CAD and FEA. Due to the versatility of NURBS, NURBS-based isogeometric analysis is mainly researched recently. NURBS basis function is widely used in CAD systems and has a lot of advantages as the basis function of FEA: a higher order continuity and an exact geometric description. Isogeometric analysis embraces the whole field of computational mechanics such as conventional structural analysis, fluid–structure interactions, optimizations, contacts, dynamics, and so on.

2.2. Trimmed surface analysis

NURBS-based isogeometric analysis has been applied to various fields of computational mechanics with substantial benefits over the classical FEA. However, NURBS-based isogeometric analysis experiences severe inflexibilities when dealing with the models with complex geometries. This is due to the tensor-product nature of NURBS objects: the object should be composed of multiple patches of rectangular shape. To model the complex geometries with multiple patches is not straightforward and cumbersome task. Therefore, in CAD systems, trimming techniques are widely employed. The trimming technique represents complex geometries by cutting-off unnecessary regions from rectangular patches. This technique significantly simplifies the modeling processes of NURBS objects. A brief illustration of the trimming technique is presented in Fig. 2. Even for the simple geometry, the circular plate with two holes, 6 patches are necessary when the trimming technique is not employed. As shown in Fig. 2(a), the modeling process is simple and straightforward when the trimming technique is employed.

Since conventional NURBS-based isogeometric analysis is unable to handle the trimmed information provided from CAD systems, an inefficient multiple-patch modeling process should be performed prior to the analysis. This inefficient process is against the spirit of isogeometric analysis. To get rid of this inefficiency, trimmed surface analysis [32,34] was proposed. In this method, the NURBS surface object with trimming technique is readily analyzed by deploying trimming information exported from CAD systems.