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Efficient 3D analysis of laminate structures using ABD-equivalent material models

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ABSTRACT

Laminate composites are widely used in automotive, aerospace, and increasingly in consumer industries, due to their reduced weight and superior structural properties. However, structural analysis of complex laminate structures remains challenging. 2D finite element methods based on plate/shell theories may be accurate and efficient, but they generally do not apply to the whole structure and require identification and preprocessing of the regions where the underlying assumptions hold. Fully automated structural analysis using solid 3D elements with sufficiently high order basis functions is possible in principle, but is rarely practiced due to the significant increase in the cost of computational integration over a large number of laminate plies.

We propose a procedure to replace the original laminate by much simpler new virtual material models. These virtual material models, under the usual assumptions made in lamination theory, have the same constitutive relationship as the corresponding 2D plate model of the original laminate, but require only a small fraction of computational integration costs in 3D FEA. We describe implementation of 3D FEA using these material models in a meshfree system using second order B-spline basis functions. Finally, we demonstrate their validity by showing agreement between computed and known results for standard problems.

1. Introduction

1.1. Motivation

Laminate composites are now used widely in variety of industries, including aerospace, automobile, medical and sports [1–3]. Laminates are lightweight and stiff with customizable material properties, resulting in structures superior to those made of homogeneous materials [2–4]. High stiffness-to-weight ratio is achieved using fiber reinforced plies. These plies, when fused together under high temperature and pressure, form complex monolithic laminate parts. The fiber reinforcements, laid using techniques ranging from manual to fully automatic, are generally parallel and unidirectional and, therefore, result in plies which are anisotropic in nature. Material properties are customized by varying fiber angle within each ply, controlling the number of plies, and adding additional materials between plies such as cores and fillers. The presence of numerous plies, however, leads to complex geometry and material distribution in laminate structures, and, therefore, structural analysis of laminates by treating each ply layer individually is prohibitively expensive. The common practice is to assume that the layers are permanently fused together and ignore any fluctuation in stress–strain fields at the interfaces of layers [3–6]. These assumptions allow to approximate laminate's global behavior as that of a plate or a shell. Interlaminar stresses and strains may be significant in boundary regions and regions of discontinuities [7] where full three-dimensional and/or layered methods should be used, but the plate/shell assumptions give sufficiently accurate stress and strain estimates for regions away from those regions [8]. In this paper, we will show that the same plate/shell assumptions, when applicable, may be used within a general 3D finite element analysis to dramatically speed up the analysis procedure.

Structural analysis of laminates can be carried out using different finite element methods, and some of them are illustrated for a typical laminate part in Fig. 1. During finite element analysis (FEA), stiffness matrix $\mathbf{K}_e$ for each element must be computed, which in general form is given as [9]

$$\mathbf{K}_e = \int_{\Omega_e} \mathbf{B}^T \cdot \mathbf{Q} \cdot \mathbf{B} \, d\Omega, \tag{1}$$

where $\mathbf{B}$ is the strain–displacement matrix, $\mathbf{Q}$ is the material constitutive relation matrix, and Ω_e is the element's domain over which integration is done. Since there are numerous plies, meshing each ply independently requires a large number of elements and is, therefore, prohibitively expensive. A much smaller number

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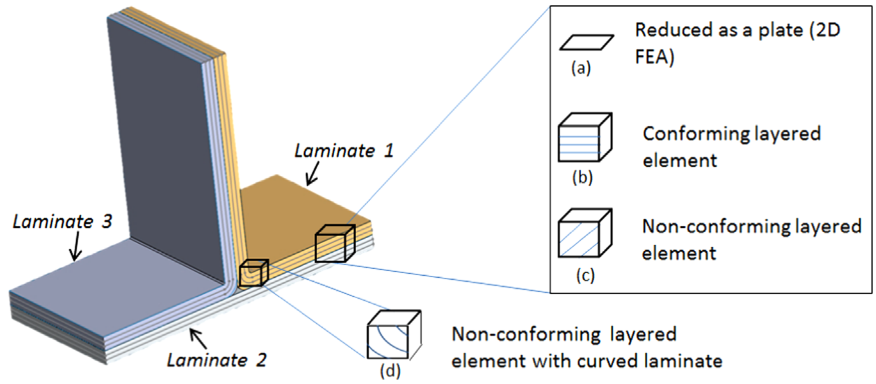

Fig. 1. Figure shows a structure made of 3 laminates analyzed using different finite element methods. (a) Plate/shell element. (b) Element of 3D conforming mesh. (c) Element of a 3D non-conforming mesh. (d) Element of a non-conforming mesh with curved laminate inside.

of elements is needed if *layered element method* [10–12] is used (Fig. 1b and c), wherein multiple plies can cross through an element. Such finite element methods (also called 3D FEA since integration domain Ω_e is 3D) may be further classified into conforming and non-conforming (or meshfree), depending on whether the finite element mesh conforms to the geometry of the laminate. Three dimensional elements may exhibit locking and ill-conditioning of stiffness matrix when used for laminates that are thin in the ply layup direction [9,13]. These problems can be alleviated or eliminated by using higher order *hierarchical*[1] basis functions [14,15].

All 3D FEA methods are computationally expensive, as the above integration has to be carried over large number of plies (tens or even hundreds). Integration is performed using quadrature rules that depend on the geometry of the element as well as the degree of the integrand, and amounts to sampling the integrand at a number of quadrature points [9]. To get an idea of the high cost of integration for laminates, we consider the layered element used in reference [10] to analyze a laminate made of 100 plies. The element used is an eight-node brick element with tri-linear basis functions, which, for a homogeneous material, is fully integrated using 2 integration points in each direction, or 8 integration points in total [10]. However, in a laminate, 8 integration points are needed for each ply, which results in a 100 fold increase for our 100-ply laminate. Since integration cost represents a significant portion of the overall solution procedure, analysis of composite laminates using layered elements is an expensive proposition.

Plate or shell assumptions reduce the computation cost and increase accuracy of FEA for laminates owing to their thin walled nature. These assumptions may lead to different *lamination theories*, where the material matrices **Q** of all the plies are replaced by the so-called ABD matrices [4,16]. The structure and the integration domain Ω_e effectively reduces to a surface (Fig. 1a), which is why this method is also called 2D FEA. However, 2D FEA is not valid in regions near boundaries and discontinuities (Fig. 1), which have significant 3D stresses and, therefore, plate/shell assumptions are invalid. In this sense, 2D FEA methods are not *general*, because such regions are common in laminate structures.

1.2. Contributions and outline

Based on the above discussion, the choice between 2D and 3D FEA amounts to a trade-off between generality and computational efficiency. We seek to develop an approach to analysis of composite laminate structures that is as general as 3D FEA and as efficient as 2D FEA when dimensional reduction makes sense.

Specifically, we propose a method to reduce the excessive cost of integration for layered elements by taking advantage of plate/shell nature of laminates, whenever such assumptions are reasonable. To this end, we have devised a procedure to obtain material models which are simpler but are equivalent to the original laminate, under the assumption made in lamination theories. We refer to these new material models as *ABD-equivalent* material models, as they result in the same ABD matrices as the original laminate and, therefore, can replace the original laminate during integration if plate/shell assumptions apply. We demonstrate the effectiveness of two such material models—a 3-ply and a graded material model—in a non-conforming FEA system using layered solid elements. We validate the two ABD-equivalent material models by using them to analyze several benchmark problems, and compare obtained results from known results. The fully implemented non-conforming FEA system uses layered solid elements with second-degree B-spline basis functions that are hierarchical in nature.

A brief outline of the paper is as follows. In Section 2, we survey the related work. Section 3 develops the concept of ABD-equivalent material models and proposes two specific examples of such models. Implementation of the proposed approach in combination with a non-conforming finite element method is described in Section 4. Its effectiveness is demonstrated using a number of benchmark problems in Section 5, followed by conclusions and future work in Section 6.

2. Related work

The finite element methods for simulating global behavior of laminate structures [16–18] may be broadly classified as a 2D or a 3D FEA. For the purposes of this paper, we only consider those methods which ignore inter-ply phenomena, but we note that the inter-layer stress–strain can be partially predicted from global deformation [4,19].

2.1. Two-dimensional FEA

Laminates usually behave as plates or shells, and are analyzed using 2D FEA. Depending on the strain field assumed in the laminate's thickness direction, different lamination theories exist [5,17,16], and can be classified as one of the following: Classical Lamination Plate Theory (CLPT), First Order Shear Deformation Theory (FSDT), or Higher Order Shear Deformation Theories (HSDTs). CLPT assumes that laminates undergo only stretching and pure bending (Fig. 2C): in-plane strains vary linearly in the thickness direction, and out-of-plane strains are absent. On the other hand,

[1] A basis function is called *hierarchical* when a higher order basis function contains all the lower order basis functions; for example, B-splines are hierarchical basis functions.