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Adaptive zooming method for the analysis of large structures with localized nonlinearities

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ABSTRACT

Simulating concrete cracking requires nonlinear modeling applied on a refined mesh if a correct evaluation of crack properties needs to be achieved. Therefore, it is rather costly and even sometimes impossible when large reinforced concrete structures are considered. Alternative solutions have therefore to be proposed. This contribution presents a structural zooming method for the simulation of large reinforced concrete structures with localized nonlinearities. Our method is based on static condensation (Guyan [1]) and provides an adaptive framework for performance-oriented use of this method in nonlinear simulations. In particular, it only simulates the behavior of nonlinear interesting zones (detected by adapted criteria). The areas where refined modeling is not required are replaced by their equivalent stiffnesses. The linearity criteria, depending on the chosen mechanical models, are also used to activate new interesting zones during the simulation. This method substantially decreases the computational cost on both presented test cases (a two-dimensional concrete beam and a three-dimensional reinforced concrete building).

1. Introduction

With recent improvements in numerical and behavior models and new features of simulation software, the complexity of models used by physicists and engineers is rising. Scalability of models and computations has become a key issue, as simulation of local small-scale phenomenon on large-scale problems seems to be the next frontier in computing. The interaction between different scales and different physics raises new questions, such as the localization and quantification of cracking risk (civil engineering, aeronautics …).

In this contribution, the specific physical phenomenon to be predicted is the cracking of large-scale concrete and reinforced concrete structures. A damage mechanics approach is used to model the behavior of concrete with the finite elements method. Simulating this phenomenon requires refined nonlinear models of the behavior of concrete (for instance, a damage mechanics approach), applied on meshes fine enough to match the physical phenomena. Those models are hardly applicable to large-scale civil engineering structures with a mesh fine enough to adequately represent cracking. However, concrete cracking is a phenomenon that is localized on large structures, and therefore, realizing a fully refined simulation of a large structure is unnecessary to obtain data on cracking: a method for simulating localized nonlinearities on large problems with accurate local information is required. Several techniques bring answers to this scalability need.

For instance, global–local analysis methods are mainly used in multi-scale simulations where physical phenomena appear at different scales of the problem. They allow separating the scales and therefore limit the complexity of the global problem [2]. These methods combine a coarse simulation at the large scale and a fine simulation at the small scale. Boundary conditions are exchanged between both simulations and an iterative scheme allows finding a solution satisfying both scales. Separating scales however requires the definition of a REV (Representative Elementary Volume), small enough to be considered as representative of the material behavior seen as homogenous at the large scale and large enough to represent a mean behavior at the small scale. To our knowledge, the definition of a REV for a cracked material is still under discussion. Also some variations on this methodology exist [3–6], eventually combining different models but also different numerical methods, such as Confinement-Shear-Lattice

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[7] or discrete elements [8]. A zooming method using global–local methodology has been introduced in 2003 by Haidar et al. to simulate cracking of concrete [9].

Another approach to solve large structural problems in limited computation time is the use of parallel computing and domain decomposition method, that decompose a large problem in smaller subproblems solved independently [10,11]. Those methods divide the computing load to take advantage of parallel architectures and solve problems faster. However, they do not affect the global computing load, and will not allow finer simulations for a given computing power [12–14].

Model reduction methods, and zooming method which derived from them, are used when complex phenomena operate in a localized manner on a large dimension problem. These methods aim to simplify at most the modeling of the areas that present no complex phenomenon, and focus the computational effort where it is the most necessary, to limit the global computing load [15]. One of the most widely used model reduction technique is orthogonal decomposition, e.g. using Karhunen–Loève expansion. It was introduced in structural mechanics by Spanos and Ghanem, based on the method previously used in fluid [16–18]. It consists in analyzing by the method of principal component analysis the mechanical responses depending on solicitations and parameters to build a predictive model. Applying this technique to a substructure allows to replace its model by a linearized model and focuses the computational effort on other substructures [19]. Possible applications include fluid mechanics, shape optimization, nuclear structure dynamic analysis [20], and stochastic structural simulation with finite elements [21]. However, those methods require previous knowledge on the localization of nonlinearities (and specifically the cracks in concrete) and are unable to detect the appearance of new cracks out of the zoomed areas.

The static condensation method, or superelements method, was introduced by Guyan [1]. It simplifies the solving of large-scale linear problems by eliminating a well-chosen subsystem of degrees of freedom. Several developments have been made on the static condensation method, such as adaptation to dynamic problems. Dynamic condensation was introduced to reduce computational cost of the dynamical analysis of large structures [15,22]. Applications are found in various fields, such as earthquake engineering, or vehicle suspension-tires systems simulation [23].

The first structural zooming method was introduced by Hirai et al. [24,25]. It combines static condensation with the reanalysis methods introduced by Wang and Pilkey [26] and remeshing techniques to locally improve the quality of numerical simulation. Starting with a relatively coarse mesh, it can be locally refined (on a substructure) with a remeshing technique. A new stiffness is calculated with the new mesh (and degrees of freedom) on the substructure, which is then condensed on the coarse mesh by Guyan reduction, and reintroduced in the global problem. Solving the problem gives the condensed solution on the original mesh, and the local subproblem can be solved from there. Only the stiffness of the area of interest is then condensed in this method. This method can also be used to combine different models at different scales, such as beam theory with nonlinear mechanics in aeronautical structural mechanics [27].

Several of the previously cited methods allow limiting the computational cost on problems similar to the cracking prediction on large concrete structures. However, all focus on obtaining the correct global behavior for the structure. Since an accurate simulation of the cracked areas is required, a new method has been developed, that focuses the computational effort on the local simulation. The proposed structural zooming method aims at reproducing at the local scale the results of a full nonlinear simulation for a fraction of the computation cost. This adaptive structural zooming method is presented in Section 2. Finally, the method is applied to a bending concrete beam and a reinforced concrete building under pressure.

2. Structural zooming method

2.1. Guyan condensation method: general principle

This paragraph describes the first-level Guyan reduction technique used in this zooming method for structural mechanics. The discretized problem of continuum mechanics to be solved at each area is formulated as follows:

$$KU = F \tag{1}$$

In the field of mechanics, with n degrees of freedom, K represents the structural stiffness matrix ($K \in \mathbb{R}^{n,n}$), U the nodal displacement ($U \in \mathbb{R}^n$), and F the equivalent external nodal force vector ($F \in \mathbb{R}^n$). All stiffness matrices are supposed invertible, that simplifies the formulation, and computational methods allow bringing properly-formulated problems back to this case. In particular, in the simulation software used in this study, Cast3M [28], boundary conditions are treated using the double Lagrange multiplier method: this method allows to keep invertible stiffness matrices that contains the boundary conditions (of the form $CU_i = U_d$) if the Lagrange multiplier vectors are added to the degrees of freedom U of the system [29].

By using two subdomains (s and m), the system becomes

$$F = \begin{pmatrix} F_s \\ F_m \end{pmatrix},\ U = \begin{pmatrix} U_s \\ U_m \end{pmatrix},\ K = \begin{pmatrix} K_{s,s} & K_{s,m} \\ K_{m,s} & K_{m,m} \end{pmatrix} \tag{2}$$

with F_s, $U_s \in \mathbb{R}^p$, $F_m, U_m \in \mathbb{R}^q$, $K_{s,s} \in \mathbb{R}^{p,p}$, $K_{s,m} \in \mathbb{R}^{p,q}$, $K_{m,s} \in \mathbb{R}^{q,p}$, $K_{m,m} \in \mathbb{R}^{q,q}$, $n = p+q$.

The system is rewritten to solve the equation of the m subdomain, using the “condensed”, or “equivalent stiffness” $\hat{K}$ of both subdomains [1]

$$U_s = K_{s,s}^{-1}(F_s - K_{s,m}U_m) \tag{3}$$

$$\hat{K}U_m = \hat{F} \tag{4}$$

$$\begin{cases} \hat{K} = K_{m,m} - K_{m,s}K_{s,s}^{-1}K_{s,m} \\ \hat{F} = F_m - K_{m,s}K_{s,s}^{-1}F_s \end{cases} \tag{5}$$

Eq. (4) is called the condensed problem. Considered independently, this q-dimensional condensed problem ($q < n$) is simplified and therefore faster to solve than the initial n-dimensional system. It solves the problem for subdomain m, and its result allows deducing the solution for subdomain s from Eq. (3). A term of “equivalent loading” ($-K_{m,s}K_{s,s}^{-1}F_s$) appears in the condensed problem (Eq. 5). It contains the influence of the external forces F_s applied on the s subdomain over the m subdomain: the subdomains have therefore been decoupled in terms of degrees of freedom but not in terms of forces and stiffnesses.

2.2. Proposed approach

This method provides a framework for an adaptive, local-scale oriented use of a two-level Guyan condensation method when solving nonlinear structural problems. It follows several steps. Fig. 1 describes the different steps of the method applied to a 2D plate problem decomposed in 6 areas. It illustrates the general principle of the method on a small academic problem. Two phases