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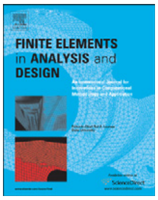

Engineering analysis in imprecise geometric models

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ABSTRACT

Holes, gaps, dangling boundaries and other imperfections of the geometric models preclude direct application of traditional engineering analysis tools. In such cases geometric inaccuracies have to be removed using a geometry "healing" (repair) procedure which results in a valid solid model. Repair procedure applied to the geometric model is computationally expensive and often requires human intervention and supervision. On the other hand, the repair procedure applied to the surface meshes derived from the boundaries of a geometric model may negatively affect the quality of the Finite Element mesh whose construction follows the repair procedure.

In this paper we describe a novel numerical technique that enables engineering analysis in imprecise geometric models without reconstructing a valid solid model. At the heart of the proposed method lies a modified geometrically adaptive integration technique. It uses a hybrid geometric model, that consists of a hierarchical space decomposition, boundary representation (B-rep) and distance fields. Hierarchical space decomposition helps to resolve the geometric imperfections, while the original geometric model is used to allocate the integration points in the boundary (geometry) cells. The proposed method uses solution structures that combine together the distance fields to the geometric boundaries, boundary conditions and basis functions to enforce the prescribed boundary conditions.

Our approach has been verified on several numerical examples. Our numerical experiments confirm high reliability of the proposed engineering analysis approach for a wide range of geometric imperfections. Despite that the paper presents 2D examples the proposed approach can be generalized in 3D.

1. Introduction

Engineering analysis is one of the important components of every design process. It accelerates product development, ensures safety and durability of the products. Currently, life of almost every product starts from creation of a Computer Aided Design (CAD) geometric model. During design-analysis cycle geometric models are modified to accommodate the changes dictated by the analysis results, shape optimization, etc. In many cases extensive editing of CAD models and their translation from one CAD system to another result in geometric inaccuracies and inconsistencies [1]. There are two major sources of errors in geometric models: inconsistencies and inaccuracies presented in the geometric representation; and geometric errors due to conversion from one CAD system to another [2]. Inconsistencies and inaccuracies in a geometric representation such as, for example, free edges (edges connected to one face), non-manifold boundaries, sliver faces and inaccurately computed intersections of the surface patches are accumulated during editing of geometric models. Some of these geometric imperfections are shown in Fig. 1(a). Acquired and reverse engineered geometric models which are represented as a "polygonal soup" [3] may contain holes in their boundaries due to the missing (non-scanned) pieces (Fig. 1(b)). Incorrectly chosen tolerances in the stereolithography (STL) geometric models [4] may produce either gaps or non-manifold boundaries. Conversion of the geometric models from one CAD system to another may result in loss of semantics information and numerical precision. Because different CAD systems use different geometric tolerances and geometric algorithms, loss of numerical precision often causes appearance of gaps, misaligned boundaries, and topological inconsistencies in geometric model which undefines the notion of "inside" and "outside".

Despite that these geometric imperfections can be very small and almost invisible, they often prevent Finite Element (FE) meshing and direct application of engineering analysis methods based on the Finite Element paradigm. The need to perform the analysis in the presence of geometric inaccuracies and imperfections led to the development of various geometry healing (repair) techniques and tools [5] that can be classified as the volumetric [6], surface [7–11] and hybrid [12] healing techniques. Volumetric techniques [6] represent the surface of the geometric model in the

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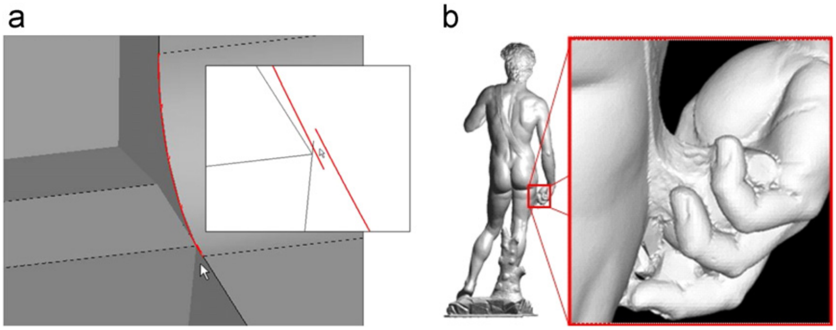

Fig. 1. (a) Inconsistencies and inaccuracies in a geometric representation such as, for example, free edges (edges connected to one face), non-manifold boundaries, and inaccurately computed intersections of the surface patches are accumulated during editing of geometric models. (b) Noisy triangulated surface of the Michelangelo's David statue contains holes due to the limitations of the laser scanning technology.

volumetric domain and apply the healing methods to the volumetric model. After fixing geometric imperfections, the volumetric model is transformed back to the surface model using either marching cubes or dual contouring algorithms [13,14]. Application of volumetric healing methods produces manifold boundaries, but these methods often result in over tessellated geometric boundaries. Surface-based geometry repair techniques perform local modifications of the original model at the places where the geometric errors and inconsistencies have been detected. These techniques are directly applied to the tessellated (usually triangulated) boundary [8]. A surface repair method applied directly on a Boundary representation (B-rep) geometric model was proposed in [9]. Geometric computations are used to determine intersections of the neighboring surface patches, projecting and inserting boundary edges into geometric faces [12]. Gaps between tessellated surface patches are removed by using stitching [10] or zippering [15] algorithms. Surface healing methods often generate small or skewed triangles that lead to poor FE meshes. Despite that automated geometry repair tools [16] have been proposed, in many real-world cases they usually require human supervision and intervention [17]. Geometry repair also requires careful analysis of a variety of special cases of possible geometric defects and the ways how they can be removed [18,19,11]. Recently, knowledge based and learning based geometry healing algorithms were proposed in [20,21]. An adaptive Cartesian mesh generation approach was proposed in [17]. It is based on volumetric geometry healing and works well for small geometric imperfections.

In this paper we propose and demonstrate a novel numerical technique that enables engineering analysis in imprecise geometric models without reconstructing a valid geometric model. It is based on the solution structure method [22–24] and modified geometrically adaptive adaptive integration technique which will be presented in Section 3.2. The proposed engineering analysis method is using the boundaries of the *original* geometric model to enforce the prescribed boundary conditions. It is also used to place the integration nodes. The proposed method requires computations of a Point Membership Classification (PMC) at the nodes of hierarchical space decomposition. These PMC values are then used by a geometrically adaptive integration [25,23] to place integration or collocation points at which the governing equation is enforced. For the valid geometric models PMC can be computed by ray casting/stabbing [6] or by computing the sign of a signed distance to the boundaries of a geometric model, etc. However, imprecisions in the geometric model prohibit direct computation of PMC. Also, as we pointed out earlier, they make the notion of ‘‘inside’’ and ‘‘outside’’ undefined. To compute PMC values for imprecise geometric models we adopt an approach similar to the one described in [6,26,5]. The main difference between the proposed technique and traditional approaches lies in the fact that our method involves the repair of neither geometrically conforming meshes nor the original geometric model. Instead, it uses a composite geometric representation to resolve geometric inaccuracies and allocate the integration points without reconstructing a valid solid model.

Besides the governing equation, the solution method has to provide the means of satisfying the prescribed boundary conditions. The salient feature of the solution structure method is the exact treatment of the specified boundary conditions by using *solution structures*—expressions that combine boundary conditions, basis functions and the functions vanishing on the geometric boundaries. The method is essentially meshfree and does not require construction of spatial meshes that conform to the shape of a geometric model. Also, it does not restrict the choice of the basis functions used for solution approximation or the solution method. Refs. [23,22,27,28] report successful implementation of the solution structure method with classical and trigonometric polynomials, as well as B-splines defined over uniform and non-uniform Cartesian grids. Providing exact treatment to the boundary conditions, the method can employ any suitable solution technique to compute numerical values of the degrees of freedom in the solution. And last, but not least, the solution structure method enables complete automation of the solution procedure [29,23,24].

1.1. Outline

The rest of the paper is organized as follows: Section 2 provides basic information on the solution structure method. It describes how the solution structures are constructed and used to satisfy the given boundary conditions. It also explains how the standard solution techniques have to be modified in order to accommodate solution structures. In Section 3 we describe a geometrically adaptive integration technique and its modification for handling the imprecise geometric models. The proposed engineering analysis approach is tested on a number of benchmark problems which are presented in Section 4. Section 5 summarizes advantages and weaknesses of the proposed approach and discusses the future directions. In this paper we present solutions of 2D example problems, but the proposed method can be easily generalized in 3D.

2. Solution structure method

2.1. Boundary value problem and its solution structure

The Solution Structure Method was originated by Kantorovich in 1950s. He proposed a simple technique to satisfy Dirichlet